Zadatak 12. Površina trokuta $\triangle ABC$ jednaka je $25\ \text{cm}^2$, umnožak ab duljina stranica a i b iznosi $58\ \text{cm}^2$, a za kutove α i β vrijedi $\sin\alpha = \cos\beta$. Odredi duljine stranica i kutove trokuta.

Rješenje.

$$P = 25\,\text{cm}^2$$
$$a \cdot b = 58\,\text{cm}^2$$
$$\sin\alpha = \cos\beta$$

$$a,\ b,\ c,\ \alpha,\ \beta,\ \gamma = ?$$
$$\sin\gamma = \frac{2P}{ab}, \quad \gamma = 59°33'$$
$$\alpha + \beta = 180° - \gamma = 120°27' \implies \beta = 120°27' - \alpha$$
$$\sin\alpha = \cos(120°27' - \alpha)$$
$$\sin\alpha = \cos 120°27' \cos\alpha + \sin 120°27' \sin\alpha$$
$$\sin\alpha = -0.50679\cos\alpha + 0.96207\sin\alpha$$
$$0.13793\sin\alpha = -0.50679\cos\alpha \quad / : 0.13793\cos\alpha$$
$$\text{tg}\,\alpha = -3.6743015$$
$$\alpha = 105°14'$$
$$\beta = 15°13'$$
$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta}$$
$$\frac{a}{\sin 105°14'} = \frac{\frac{58}{a}}{\sin 15°13'}$$
$$\frac{a}{\sin 105°14'} = \frac{58}{a\sin 15°13'}$$
$$a^2 \sin 15°13' = 58\sin 105°14'$$
$$a^2 = \frac{58\sin 105°14'}{\sin 15°13'}$$
$$a = 14.6\,\text{cm}$$
$$b = \frac{58}{14.6} \approx 4\,\text{cm}$$
$$c = \frac{a\cdot\sin\gamma}{\sin\alpha} = 13.1\,\text{cm}.$$