Zadatak 26. Duljine stranica trokuta $\triangle ABC$ u omjeru su $4:5:8$. Polumjer trokutu opisane kružnice iznosi 9 cm. Kolika je površina trokuta?

Rješenje.

$$a:b:c=4:5:8$$
$$R=9\,\text{cm}$$

$$P=?$$
$$a:b:c=4:5:8=\sin\alpha:\sin\beta:\sin\gamma$$
$$a=4x,\quad b=5x,\quad c=8x;$$
$$\sin\alpha=4y,\quad \sin\beta=5y,\quad \sin\gamma=8y;$$
$$\frac{a}{\sin\alpha}=2R \implies 4x=2\cdot 9\cdot 4y \implies x=18y$$
$$a^2=b^2+c^2-2bc\cos\alpha \qquad (\cos\alpha=\sqrt{1-\sin^2\alpha})$$
$$16x^2=25x^2+64x^2-2\cdot 5x\cdot 8x\cdot\sqrt{1-16y^2}$$
$$-73x^2=-80x^2\sqrt{1-16y^2}$$
$$73=80\sqrt{1-16y^2}$$
$$0.9125=\sqrt{1-16y^2} \quad /^2$$
$$0.832653=1-16y^2$$
$$16y^2=0.167347$$
$$y^2=0.0104591875$$
$$y=0.10227$$
$$\sin\alpha=0.4091,\quad \sin\beta=0.5113,\quad \sin\gamma=0.8182$$
$$P=2R^2\sin\alpha\sin\beta\sin\gamma=27.72\,\text{cm}^2$$