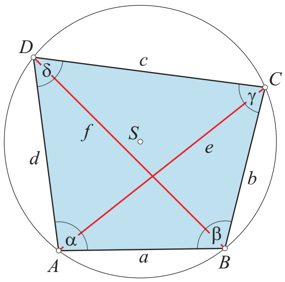
Zadatak 14. U tetivnom četverokutu je $a = 8$ cm, $b = 9\,cm$, $c = 13\,cm$, $d = 10$ cm. Izračunaj mu kutove i površinu.

Rješenje.



$a = 8\,\text{cm}$

$b = 9\,\text{cm}$

$c = 13\,\text{cm}$

$d = 10\,\text{cm}$

$\alpha,\ \beta,\ c,\ \delta,\ P = ?$

$$s = \frac{a+b+c+d}{2} = 20\,\text{cm}$$

$$P = (\text{tetivni četverokut}) = \sqrt{(s-a)(s-b)(s-c)(s-d)} = \sqrt{12\cdot 11\cdot 7\cdot 10} = 96.12\,\text{cm}^2$$

$$\alpha+\gamma = \beta+\delta = 180^\circ$$

$$f^2 = a^2+d^2-2ad\cos\alpha \qquad (1)$$

$$f^2 = b^2+c^2-2bc\underbrace{\cos\gamma}_{=-\cos\alpha} \qquad (2)$$

$$(1)-(2) \implies a^2+d^2-b^2-c^2-2\cos\alpha(ad+bc) = 0$$

$$\cos\alpha = \frac{a^2+d^2-b^2-c^2}{2(ad+bc)} \implies \alpha = 102^\circ 36' 26''$$

$$\gamma = 180^\circ - \alpha = 77^\circ 23' 34''$$

$$e^2 = a^2+b^2-2ab\cos\beta \qquad (3)$$

$$e^2 = c^2+d^2-2cd\underbrace{\cos\delta}_{=-\cos\beta} \qquad (4)$$

$$(3)-(4) \implies a^2+b^2-c^2-d^2-2\cos\beta(ab+cd) = 0$$

$$\cos\beta = \frac{a^2+b^2-c^2-d^2}{2(ab+cd)} \implies \beta = 107^\circ 52' 25''$$

$$\delta = 180^\circ - \beta = 72^\circ 7' 35''$$