Zadatak 12. Dokaži da u svakom trokutu vrijedi

$$\left(\frac{a}{b}+\frac{b}{a}\right)\cos\gamma+\left(\frac{b}{c}+\frac{c}{b}\right)\cos\alpha+\left(\frac{c}{a}+\frac{a}{c}\right)\cos\beta=3.$$

Rješenje. Prema poučku o sinusima je $\frac{a}{b}=\frac{\sin\alpha}{\sin\beta}$ i slično je za ostale količnike. Lijeva strana jednakosti postaje

$$\begin{aligned}
&\left(\frac{\sin\alpha}{\sin\beta}+\frac{\sin\beta}{\sin\alpha}\right)\cos\gamma+\left(\frac{\sin\beta}{\sin\gamma}+\frac{\sin\gamma}{\sin\beta}\right)\cos\alpha\\
&\qquad+\left(\frac{\sin\gamma}{\sin\alpha}+\frac{\sin\alpha}{\sin\gamma}\right)\cos\beta\\
&=\frac{1}{\sin\alpha}(\sin\beta\cdot\cos\gamma+\sin\gamma\cos\beta)\\
&\qquad+\frac{1}{\sin\beta}(\sin\gamma\cdot\cos\alpha+\sin\alpha\cos\gamma)\\
&\qquad\qquad+\frac{1}{\sin\gamma}(\sin\alpha\cdot\cos\beta+\sin\beta\cos\alpha)\\
&=\frac{\sin(\beta+\gamma)}{\sin\alpha}+\frac{\sin(\gamma+\alpha)}{\sin\beta}+\frac{\sin(\alpha+\beta)}{\sin\gamma}=3,
\end{aligned}$$

jer je $\sin(\beta+\gamma)=\sin(\pi-\alpha)=\sin\alpha$ i slično za ostale pribrojnike.