



**Zadatak 20.** U paralelogramu je zadan kut  $\alpha$  i udaljenosti  $m$  i  $n$  sjecišta dijagonala do strana  $a$  i  $b$ . Odredi površinu i dijagonale paralelograma.

**Rješenje.**  $\alpha, m, n$  zadano

$$\begin{aligned}
 P &= ab \cdot \sin \alpha = a \cdot 2m = b \cdot 2n \\
 \implies b &= \frac{2m}{\sin \alpha}, \quad a = \frac{2n}{\sin \alpha} \\
 P &= \frac{2n}{\sin \alpha} \cdot \frac{2m}{\sin \alpha} \cdot \sin \alpha = \frac{4mn}{\sin \alpha} \\
 e^2 &= a^2 + b^2 - 2ab \cos(180^\circ - \alpha) \\
 &= a^2 + b^2 + 2ab \cos \alpha \\
 &= \frac{4n^2}{\sin^2 \alpha} + \frac{4m^2}{\sin^2 \alpha} + 2 \frac{2n}{\sin \alpha} \frac{2m}{\sin \alpha} \cos \alpha \\
 &= \frac{4}{\sin^2 \alpha} (n^2 + m^2 + 2mn \cos \alpha) \implies e = \frac{2}{\sin \alpha} \sqrt{m^2 + n^2 + 2mn \cos \alpha} \\
 f^2 &= a^2 + b^2 - 2ab \cos \alpha = \frac{4n^2}{\sin^2 \alpha} + \frac{4m^2}{\sin^2 \alpha} - 2 \frac{2n}{\sin \alpha} \frac{2m}{\sin \alpha} \cos \alpha \\
 &= \frac{4}{\sin^2 \alpha} (n^2 + m^2 - 2mn \cos \alpha) \implies f = \frac{2}{\sin \alpha} \sqrt{m^2 + n^2 - 2mn \cos \alpha}
 \end{aligned}$$

