Zadatak 25. Trokut stranice a i kutova β i γ rotira oko stranice c. Koliko je oplošje i obujam rotacijskog tijela?

Rješenje.

$a,\ \beta,\ \gamma$ zadano $\implies \gamma$ poznato

$O,\ V =?$

$$\frac{v_a}{a} = \sin\beta \implies v_a = a\sin\beta$$

$$\frac{b}{\sin\beta} = \frac{a}{\sin\alpha} \implies b = \frac{a\sin\beta}{\sin\alpha}$$

$$\frac{c}{\sin\gamma} = \frac{a}{\sin\alpha} \implies c = \frac{a\sin\gamma}{\sin\alpha}$$

$$O = v_a\pi a + v_a\pi b = v_a\pi(a+b) = a\sin\beta\pi\left(a + \frac{a\sin\beta}{\sin\alpha}\right)$$

$$= \frac{a^2\sin\beta\pi}{\sin\alpha}(\sin\alpha + \sin\beta)$$

$$V = \frac{v_a^2\pi\cdot x}{3} + \frac{v_a^2\pi\cdot(c-x)}{3} = \frac{v_a^2\pi\cdot c}{3} = a^2\sin^2\beta\pi\frac{a\sin\gamma}{\sin\alpha} = \frac{a^3\sin^2\beta\pi\sin\gamma}{\sin\alpha}$$