

Zadatak 30. Duljina osnovnog brida pravilne trostrane piramide jednaka je a , a prikloni kut bočnog brida prema osnovici piramide iznosi α . Koliki je volumen sfere opisane ovoj piramidi?

Rješenje.

α , azadano

$R = ?$

$$\frac{v}{b} = \sin \alpha \implies v = b \sin \alpha$$

$$\frac{v}{\frac{2}{3}v_b} = \operatorname{tg} \alpha$$

$$v = \frac{2}{3}v_b \operatorname{tg} \alpha = \frac{2}{3} \cdot \frac{a\sqrt{3}}{2} \cdot \operatorname{tg} \alpha = \frac{a\sqrt{3}}{3} \operatorname{tg} \alpha$$

$$(v - R)^2 + \left(\frac{2}{3}v_b\right)^2 = R^2$$

$$\left(\frac{a\sqrt{3}}{3} \operatorname{tg} \alpha - R\right)^2 + \left(\frac{2}{3} \frac{a\sqrt{3}}{2}\right)^2 = R^2$$

$$\frac{3a^2}{9} - \frac{2a\sqrt{3}R}{3} \operatorname{tg} \alpha + R^2 + \frac{3a^2}{9} = R^2$$

$$- \frac{2a\sqrt{3}R}{3} \operatorname{tg} \alpha = - \frac{3a^2}{9} - \frac{3a^2}{9} \operatorname{tg}^2 \alpha$$

$$2\sqrt{3}R \operatorname{tg} \alpha = a - a \operatorname{tg}^2 \alpha$$

$$R = \frac{a(1 - \operatorname{tg}^2 \alpha)}{2\sqrt{3} \operatorname{tg} \alpha} = \frac{a \frac{\cos^2 \alpha - \sin^2 \alpha}{\cos^2 \alpha}}{2\sqrt{3} \frac{\sin \alpha}{\cos \alpha}} = \frac{a \cos 2\alpha}{2\sqrt{3} \sin \alpha \cos \alpha} = \frac{a \cos 2\alpha}{\sqrt{3} \sin 2\alpha} = \frac{a \operatorname{ctg} 2\alpha}{\sqrt{3}}$$

$$V = \frac{4R^3 \pi}{3} = \frac{4a^3 \operatorname{ctg}^3 2\alpha}{3\sqrt{3}} \pi$$

