

**Zadatak 31.**

Kvadrat  $ABCD$  zajednička je osnovka dviju sukladnih uspravnih piramida čiji su vrhovi s raznih strana kvadrata. Neka je duljina stranice kvadrata jednaka  $a$ , a  $\alpha$  kut jedne pobočke pri vrhu piramide. Koliki je polumjer sfere upisane ovom osmerostranom poliedru?

**Rješenje.** $\alpha, a$  zadano

$$\underline{R = ?}$$

$$\frac{a}{b} = \sin \frac{\alpha}{2} \implies b = \frac{a}{2 \sin \frac{\alpha}{2}}$$

$$\frac{a}{v_p} = \operatorname{tg} \frac{\alpha}{2} \implies v_p = \frac{a}{2 \operatorname{tg} \frac{\alpha}{2}}$$

$$v_p^2 = v^2 + \left(\frac{a}{2}\right)^2$$

$$\frac{a^2}{4 \operatorname{tg}^2 \frac{\alpha}{2}} = v^2 + \frac{a^2}{4}$$

$$v^2 = \frac{a^2}{4} \left( -1 + \frac{1}{\operatorname{tg}^2 \frac{\alpha}{2}} \right) = \frac{a^2}{4} \frac{\cos^2 \frac{\alpha}{2} - \sin^2 \frac{\alpha}{2}}{\sin^2 \frac{\alpha}{2}} = \frac{a^2}{4} \frac{\cos \alpha}{\sin^2 \frac{\alpha}{2}} \implies v = \frac{a}{2 \sin \frac{\alpha}{2}} \sqrt{\cos \alpha}$$

$$v_p \cdot R = \frac{a}{2} \cdot v$$

$$R = \frac{av}{2v_p} = \frac{\frac{a}{2} \frac{a}{2 \sin \frac{\alpha}{2}} \sqrt{\cos \alpha}}{\frac{a}{2 \operatorname{tg} \frac{\alpha}{2}}} = \frac{a \sqrt{\cos \alpha}}{2 \sin \frac{\alpha}{2} \operatorname{ctg} \frac{\alpha}{2}} = \frac{a \sqrt{\cos \alpha}}{2 \sin \frac{\alpha}{2} \frac{\cos \frac{\alpha}{2}}{\sin \frac{\alpha}{2}}} = \frac{a \sqrt{\cos \alpha}}{2 \cos \frac{\alpha}{2}}$$

