

Zadatak 22.

Udaljenost pravca od ishodišta koordinatnog sustava jednaka je $2\sqrt{5}$. Površina trokuta što ga pravac zatvara s koordinatnim osima iznosi 25 kv. jed. Odredi jednadžbu tog pravca.

Rješenje.

$$P = \frac{|m \cdot n|}{2}$$

$$25 = \frac{|m \cdot n|}{2}$$

$$|m \cdot n| = 50$$

$$|n| = \frac{50}{|m|}$$

$$\frac{x}{m} + \frac{y}{n} = 1 \quad / \cdot mn$$

$$nx + my - mn = 0 \implies A = n, B = m, C = -mn$$

$$T(x_0, y_0) = T(0, 0)$$

$$d = \frac{|Ax_0 + By_0 + C|}{\sqrt{A^2 + B^2}}$$

$$2\sqrt{5} = \frac{|n \cdot 0 + m \cdot 0 - mn|}{\sqrt{n^2 + m^2}}$$

$$|mn| = 2\sqrt{5}\sqrt{n^2 + m^2}$$

$$50 = 2\sqrt{5}\sqrt{\left(\frac{50}{|m|}\right)^2 + m^2}$$

$$\frac{25}{\sqrt{5}} = \sqrt{\left(\frac{50}{|m|}\right)^2 + m^2} \quad /^2$$

$$\frac{625}{5} = \frac{2500}{m^2} + m^2 \quad / \cdot m^2$$

$$m^4 - 125m^2 + 2500 = 0$$

$$m_{1,2} = \frac{125 \pm \sqrt{15625 - 10000}}{2} = \frac{125 \pm \sqrt{5625}}{2} = \frac{125 \pm 75}{2}$$

$$m_1 = \frac{125 - 75}{2} = 25 \implies m_{1,2} = \pm 5, \quad n_{1,2} = \pm 10$$

$$m_2 = \frac{125 + 75}{2} = 100 \implies m_{3,4} = \pm 10, \quad n_{3,4} = \pm 5$$

Četiri rješenja:

$$\frac{x}{\pm 5} + \frac{y}{\pm 10} = 1,$$

$$\frac{x}{\pm 10} + \frac{y}{\pm 5} = 1$$

