

Zadatak 26. Dana su dva susjedna vrha kvadrata, $A(-1, 4)$ i $B(2, 0)$. Odredi jednadžbe stranica tog kvadrata.

Rješenje.

$$\begin{aligned} a \quad (AB) \quad \dots \quad y - 4 &= \frac{0 - 4}{2 + 1}(x + 1) \\ y - 4 &= -\frac{4}{3}(x + 1) \\ y &= -\frac{4}{3}x - \frac{4}{3} + 4 \\ y &= -\frac{4}{3}x + \frac{8}{3} \end{aligned}$$

$$\begin{aligned} b \quad \dots \quad b \perp a, \quad B \in b \\ b \perp a \implies k_b = -1 \frac{1}{k_a} = -1 \frac{1}{-\frac{4}{3}} &= \frac{3}{4} \\ B \in b \implies y - 0 &= \frac{3}{4}(x - 2) \\ y &= \frac{3}{4}x - \frac{3}{2} \end{aligned}$$

$$\begin{aligned} d \quad \dots \quad d \perp a, \quad A \in d \\ d \perp a \implies k_d = -1 \frac{1}{k_a} = -1 \frac{1}{-\frac{4}{3}} &= \frac{3}{4} \\ A \in d \implies y - 4 &= \frac{3}{4}(x + 1) \\ y &= \frac{3}{4}x + \frac{3}{4} + 4 \\ y &= \frac{3}{4}x + \frac{19}{4} \end{aligned}$$

$$\begin{aligned} C \in b \quad d(B, C) &= d(A, B), \quad C\left(x_1, \frac{3}{4}x_1 - \frac{3}{2}\right) \\ D \in d \quad d(A, D) &= d(A, B), \quad D\left(x_2, \frac{3}{4}x_2 + \frac{19}{4}\right) \\ d(A, B) &= \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2} = \sqrt{(2 + 1)^2 + (0 - 4)^2} = \sqrt{9 + 16} = 5 \\ d(B, C) &= \sqrt{(x_C - x_B)^2 + (y_C - y_B)^2} \\ 5 &= \sqrt{(x_C - 2)^2 + \left(0 - \frac{3}{4}x_1 + \frac{3}{2}\right)^2} / 2 \\ 25 &= x_C^2 - 4x_C + 4 + \frac{9}{16}x_C^2 - \frac{9}{4}x_C + \frac{9}{4} \\ \frac{25}{16}x_C^2 - \frac{25}{4}x_C - \frac{75}{4} &= 0 \quad / \cdot 16 \\ 25x_C^2 - 100x_C - 300 &= 0 \quad / : 25 \\ x_C^2 - 4x_C - 12 &= 0 \end{aligned}$$

$$(x_C)_{1,2} = \frac{4 \pm \sqrt{16 + 48}}{2} = \frac{4 \pm 8}{2}$$

$$(x_C)_1 = 6, \quad (x_C)_2 = -2$$

$$(y_C)_1 = \frac{3}{4} \cdot 6 - \frac{3}{2} = 3 \implies C_1(6, 3)$$

$$(y_C)_2 = \frac{3}{4} \cdot (-2) - \frac{3}{2} = -3 \implies C_2(-2, -3)$$

$$d_1 \quad \dots \quad d_1 \| a \implies k_{d_1} = -\frac{4}{3}, \quad C_1 \in d_1$$

$$\implies y - 3 = -\frac{4}{3}(x - 6) \quad / \cdot 3$$

$$3y - 9 = -4x + 24$$

$$4x + 3y + 33 = 0$$

$$d_2 \quad \dots \quad d_2 \| a \implies k_{d_2} = -\frac{4}{3}, \quad C_2 \in d_1$$

$$\implies y + 3 = -\frac{4}{3}(x + 2) \quad / \cdot 3$$

$$3y + 9 = -4x - 8$$

$$4x + 3y + 17 = 0$$