Zadatak 32. Odredi jednadžbe stranica kvadrata kojem je točka $S(1,6)$ sjecište dijagonala, a točke $M(4,9)$ i $N(-5,4)$ pripadaju njegovim stranicama $\overline{AB}$, odnosno $\overline{BC}$.

Rješenje.

$S(1,6)$

$M(4,9)$

$N(-5,4)$

$a \ \ldots \ a = AB, \ M \in a$

$y - 9 = k(x-4) \implies kx - y - 4k + 9 = 0 \ \ldots \ a$

$B \ \ldots \ a = BC, \ N \in b, \ b \perp a \implies k_b = -\frac{1}{k}$

$y - 4 = -\frac{1}{k}(x+5) \implies x + ky - 4k + 5 = 0 \ \ldots \ b$

$d(S,a) = d(S,b)$

$|k \cdot 1 + (-1) \cdot 6 - 4k + 9| = |1 \cdot 1 + k \cdot 6 - 4k + 5|$

$|-3k + 3| = |6 + 2k|$

$-3k + 3 < 0 \implies k > 1$

$6 + 2k < 0 \implies k < -\frac{1}{3}$

	$\langle -\infty, -\frac{1}{3} \rangle$	$\langle -\frac{1}{3}, 1 \rangle$	$\langle 1, \infty \rangle$
$-3k+3$	$+$	$+$	$-$
$6+2k$	$+$	$-$	$-$

$k \in \langle -\infty, -\frac{1}{3} \rangle \cup \langle 1, \infty \rangle$

$\langle -\frac{1}{3}, 1 \rangle$

$-3k + 3 = 6 + 2k$

$3k - 3 = 6 + 2k$

$5k = -3$

$k = 9$

$k = -\frac{3}{5}$

1) $k = -\frac{3}{5}$

$a \ \ldots \ y - 9 = -\frac{3}{5}(x-4) \quad / \cdot 5$

$5y - 45 = -3x + 12$

$3x + 5y - 57 = 0$

$y = -\frac{3}{5}x - \frac{57}{5}$

$b \ \ldots \ y - 4 = -\frac{1}{-\frac{3}{5}}(x+5)$

$y - 4 = \frac{5}{3}(x+5) \quad / \cdot 3$

$3y - 12 = 5x + 25$

$5x - 3y + 37 = 0$

$y = \frac{5}{3}x + \frac{37}{3}$

$\{B\} = a \cap b \quad \ldots$

$$\left.\begin{aligned} 3x + 5y - 57 &= 0 \quad / \cdot 5 \\ 5x - 3y + 37 &= 0 \quad / \cdot (-3) \end{aligned}\right\} +$$

$$34y = 396 \implies y_B = \frac{198}{17}$$

$$3 \cdot x + 5 \cdot \frac{198}{17} - 57 = 0 \quad / \cdot 17$$

$$51x + 990 - 969 = 0$$

$$51x = -21$$

$$x = -\frac{21}{51} \implies B\left(-\frac{21}{51}, \frac{198}{17}\right)$$

S je polovište od $\overline{BD}$ pa vrijedi:

$$x_D = 2 \cdot x_S - x_B = 2 \cdot 1 + \frac{21}{51}$$

$$x_D = \frac{123}{51}$$

$$y_D = 2 \cdot y_S - y_B = 2 \cdot 6 - \frac{198}{17}$$

$$y_D = \frac{6}{17} \implies D\left(\frac{123}{51}, \frac{6}{17}\right)$$

c $\ldots$ $D \in c, \quad k_c = k = -\frac{3}{5}$

$$y - \frac{6}{17} = -\frac{3}{5}\left(x - \frac{123}{51}\right) \quad / \cdot 255$$

$$255y - 90 = -153x + 369$$

$$153x + 255y - 459 = 0 \quad / : 51$$

$$3x + 5y - 9 = 0$$

d $\ldots$ $D \in d, \quad k_d = -\frac{1}{k} = \frac{5}{3}$

$$y - \frac{6}{17} = \frac{5}{3}\left(x - \frac{123}{51}\right) \quad / \cdot 153$$

$$y - \frac{6}{17} = \frac{5}{3}\left(x - \frac{123}{51}\right) \quad / \cdot 153$$

$$153y - 54 = 255x - 615$$

$$255x - 153y - 54 = -615$$

$$153x + 255y - 561 = 0 \quad / : 51$$

$$3x + 5y - 11 = 0$$

2) $k = 9$

$a \ \ldots \ y - 9 = 9(x - 4)$

$y - 9 = 9x - 36$

$9x - y - 27 = 0$

$b \ \ldots \ y - 4 = -\frac{1}{9}(x + 5) \quad / \cdot 9$

$9y - 36 = -x - 5$

$x + 9y - 31 = 0$

$\{B\} = a \cap b \ \ldots$

$9x - y - 27 = 0 \quad / \cdot 9$

$x + 9y - 31 = 0 \quad / \cdot (-9)$

$81x - 9y - 243 = 0 \quad / \cdot 9$

$x + 9y - 31 = 0 \quad / \cdot (-9)$ $\Big\}+$

$82x - 274 = 0 \implies x_B = \frac{137}{41}$

$\frac{137}{41} + 9y - 31 = 0 \quad / \cdot 41$

$137 + 369y - 1271 = 0$

$369x = 1134$

$y_B = \frac{126}{41} \implies B\left(\frac{137}{41}, \frac{126}{41}\right)$

S je polovište od $\overline{BD}$ pa vrijedi:

$x_D = 2 \cdot x_S - x_B = 2 \cdot 1 - \frac{137}{41} = -\frac{55}{41}$

$x_D = -\frac{55}{41}$

$y_D = 2 \cdot y_S - y_B = 2 \cdot 6 - \frac{126}{41}$

$y_D = \frac{366}{41} \implies D\left(-\frac{55}{41}, \frac{366}{41}\right)$

$c \ \ldots \ D \in c, \ k_c = k = 9$

$y - \frac{366}{41} = 9\left(x + \frac{55}{41}\right) \quad / \cdot 41$

$41y - 366 = 369x + 495$

$369x - 41y + 861 = 0 \quad / : 41$

$9x - y + 21 = 0$

$d \ \ldots \ D \in d, \ k_d = -\frac{1}{k} = -\frac{1}{9}$

$y - \frac{366}{41} = -\frac{1}{9}\left(x + \frac{55}{41}\right) \quad / \cdot 369$

$$369y - 3294 = -41x - 55$$
$$41x + 369y - 3239 = 0 \quad / : 41$$
$$x + 9y - 79 = 0$$