

Zadatak 34. Stranice trokuta ABC pripadaju pravcima $x + 3y - 2 = 0$, $x - y + 2 = 0$ i $3x + y - 14 = 0$. Odredi ortocentar ovog trokuta.

Rješenje.

$$a \dots x + 3y - 2 = 0 \implies y = -\frac{1}{3}x + \frac{2}{3}$$

$$b \dots x - y + 2 = 0 \implies y = x + 2$$

$$c \dots 3x + y - 14 = 0 \implies y = -3x + 14$$

$$\{C\} = a \cap b \dots -\frac{1}{3}x + \frac{2}{3} = x + 2 \quad / \cdot 3$$

$$-x + 2 = 3x + 6$$

$$-4x = 4$$

$$x = -1$$

$$y = -1 + 2$$

$$y = 1 \implies C(-1, 1)$$

$$\{B\} = a \cap c \dots -\frac{1}{3}x + \frac{2}{3} = -3x + 14 \quad / \cdot 3$$

$$-x + 2 = -9x + 42$$

$$8x = 40$$

$$x = 5$$

$$y = -3 \cdot 5 + 14$$

$$y = -1 \implies B(5, -1)$$

$$\{A\} = b \cap c \dots x + 2 = -3x + 14 \quad / \cdot 3$$

$$4x = 12$$

$$x = 3$$

$$y = 3 + 2$$

$$y = 5 \implies A(3, 5)$$

$$v_c \dots v_c \perp c, C(-1, 1) \in v_c$$

$$k_{v_c} = -\frac{1}{k_c} = -\frac{1}{-3} = \frac{1}{3}$$

$$y - 1 = \frac{1}{3}(x + 1)$$

$$y - 1 = \frac{1}{3}x + \frac{1}{3}$$

$$y = \frac{1}{3}x + \frac{4}{3}$$

$$v_a \dots v_a \perp a, A(3, 5) \in v_a$$

$$k_{v_a} = -\frac{1}{k_c} = -\frac{1}{-\frac{1}{3}} = 3$$

$$y - 5 = 3(x - 3)$$

$$y - 5 = 3x - 9$$

$$y = 3x - 4$$

$$\{H\} = v_a \cap v_c \dots 3x - 4 = \frac{1}{3}x + \frac{4}{3} \quad / \cdot 3$$

$$9x - 12 = x + 4$$

$$8x = 16$$

$$x = 2$$

$$y = 3 \cdot 2 - 4$$

$$y = 2 \implies H(2, 2)$$

Vrhovi trokuta su točke $A(-1, 1)$, $B(5, -1)$, $C(3, 5)$, a ortocentar je $H(2, 2)$.