Zadatak 6. Dijagonala kvadrata $ABCD$ pripada pravcu $4x+3y-12=0$. Ako je točka $A(-1,-3)$ jedan vrh, odredi ostala tri vrha kvadrata.

Rješenje.

$$d_1 \ \ldots \ 4x+3y-12=0$$
$$A(-1,-3)$$
$$A \notin d_1 \ \text{jer} \ 4\cdot(-1)+3\cdot(-3)-12=-25\neq 0$$

Tada je $d(A,d_1)=\dfrac{d}{2}$ pa vrijedi:

$$\frac{d}{2}=\frac{|4\cdot(-1)+3\cdot(-3)-12|}{\sqrt{16+9}}$$
$$\frac{d}{2}=\frac{25}{5}$$
$$\frac{d}{2}=5 \implies \boxed{d=10}$$
$$d_2\perp d_1 \implies k_{d_2}=-\frac{1}{k_{d_1}}$$
$$k_{d_1} \ \ldots \ 3y=-4x+12 \quad /\,3$$
$$y=-\frac{4}{3}+4 \implies k_{d_1}=-\frac{4}{3}, \quad k_{d_2}=\frac{3}{4}$$
$$\{A\}\in d_2 \implies y+3=\frac{3}{4}(x+1)$$
$$\boxed{y=\frac{3}{4}x-\frac{9}{4} \ \ldots \ d_2}$$
$$\{S\}=d_1\cap d_2 \ \ldots \ -\frac{4}{3}x+4=\frac{3}{4}x-\frac{9}{4} \quad /\cdot 12$$
$$-16x+48=9x-27$$
$$25x=75$$
$$\boxed{x=3}$$
$$y=-\frac{4}{3}\cdot 3+4$$
$$\boxed{y=0} \implies \boxed{S(3,0)} \ \text{(sjecište dijagonala)}$$

S je polovište od $\overline{AC}$

$$x_C=2x_S-x_A=2\cdot 3+1=7$$
$$y_C=2y_S-y_A=2\cdot 0+3=3 \implies \boxed{C(7,3)}$$

S je polovište od $\overline{BD}$, $B, D \in d_1$

$$B\left(x_1, -\frac{4}{3}x_1 + 4\right)$$
$$D\left(x_2, -\frac{4}{3}x_2 + 4\right)$$

$$x_S = \frac{x_B + x_D}{2}$$
$$y_S = \frac{y_B + y_D}{2}$$

$$3 = \frac{x_1 + x_2}{2} \quad \Big/ \cdot 2$$
$$0 = \frac{-\frac{4}{3}x_1 + 4 - \frac{4}{3}x_2 + 4}{2} \quad \Big/ \cdot 2$$

$$6 = x_1 + x_2 \implies x_2 = 6 - x_1 \qquad (1)$$
$$0 = -\frac{4}{3}x_1 - \frac{4}{3}x_2 + 8 \qquad (2)$$

(1) uvrstimo u (2) :

$$-\frac{4}{3}x_1 - \frac{4}{3}(6 - x_1) + 8 = 0 \quad \Big/ \cdot 3$$
$$-3x_1 - 4(6 - x_1) + 24 = 0$$
$$-3x_1 - 24 + 4x_1 + 24 = 0$$
$$x_1 = 0 \implies x_2 = 6$$
$$y_1 = 4, \quad y_2 = -\frac{4}{3} \cdot 6 + 4 = -4$$
$$\implies B(0, 4), \quad D(6, -4)$$