Zadatak 14. Trokut ABC je jednakokračan, njegova je osnovica dužina $\overline{AB}$, $A(-1, -2)$, $B(7, 4)$. Duljina visine na osnovicu trokuta jednaka je 5. Odredi koordinate vrha C.

Rješenje. Trokut je jednakokračan.

$$A(-1, -2)$$
$$B(7, 4)$$
$$\overline{AB} \text{ je osnovica}$$
$$\underline{v_c = 5}$$
$$C = ? \quad C(x_0, y_0)$$

$$c = AB \ \dots \ y + 2 = \frac{4 + 2}{7 + 1}(x + 1)$$
$$y + 2 = \frac{3}{4}(x + 1) \quad / \cdot 4$$
$$3x - 4y - 5 = 0$$
$$d(C, c) = v_c$$
$$\frac{|3 \cdot x_0 + (-4) \cdot y_0 - 5|}{\sqrt{3^2 + 4^2}} = 5$$
$$\frac{|3x_0 - 4y_0 - 5|}{5} = 5 \quad / \cdot 5$$
$$|3x_0 - 4y_0 - 5| = 25 \qquad\qquad (*)$$

Trokut je jednakokračan pa točka C leži na simetrali dužine AB. Neka je P polovište stranice $\overline{AB}$ tada vrijedi:

$$P\left(\frac{x_A + x_B}{2}, \frac{y_A + y_B}{2}\right) \implies P\left(\frac{-1 + 7}{2}, \frac{-2 + 4}{2}\right) \implies P(3, 1)$$

Točka P je na simetrali stranice $\overline{AB}$, $P \in s_{AB}$ te je $s_{AB} \perp AB$ pa vrijedi:

$$k_s = -\frac{1}{k_c} = -\frac{1}{\frac{3}{4}} = -\frac{4}{3}$$

$$s_{AB} \ \dots \ y - 1 = -\frac{4}{3}(x - 3)$$
$$y = -\frac{4}{3}x + 5$$
$$\implies y_0 = -\frac{4}{3}x_0 + 5$$

Zadnje dobiveno uvrstimo u $(*)$:

$$\left|3x_0 - 4\left(-\frac{4}{3}x_0 + 5\right) - 5\right| = 25$$
$$\left|3x_0 + \frac{16}{3}x_0 - 20 - 5\right| = 25$$
$$\left|\frac{25}{3}x_0 - 25\right| = 25$$

1) $\dfrac{25}{3}x_0 - 25 = -25$

$\dfrac{25}{3}x_0 = 0$

$x_0 = 0$

$y_0 = -\dfrac{4}{3} \cdot 0 + 5 = 5$

$\implies$ $\boxed{C_1(0, 5)}$

2) $\dfrac{25}{3}x_0 - 25 = 25$

$\dfrac{25}{3}x_0 = 50 \qquad \Big/ \cdot \dfrac{3}{25}$

$x_0 = \dfrac{50 \cdot 3}{25}$

$x_0 = 6$

$y_0 = -\dfrac{4}{3} \cdot 6 + 5 = -3$

$\implies$ $\boxed{C_2(6, -3)}$