Zadatak 15. Na pravcu $x - 2y + 8 = 0$ odredi točku jednako udaljenu od točke $T(8,3)$ i pravca $3x + 4y - 11 = 0$.

Rješenje.

$$p \;\ldots\; x - 2y + 8 = 0 \implies y = \frac{1}{2}x + 4$$

$$T(8,3)$$

$$q \;\ldots\; 3x + 4y - 11 = 0 \implies y = -\frac{3}{4}x + \frac{11}{4}$$

$$P = ?$$

$$p \in p \implies P\left(x_0, \frac{1}{2}x_0 + 4\right)$$

$$d(T,P) = d(P,q)$$

$$\sqrt{(x_T - x_p)^2 + (y_T - y_P)^2} = \frac{|Ax_P + By_P + C|}{\sqrt{A^2 + B^2}}$$

$$\sqrt{(8 - x_0)^2 + (3 - \tfrac{1}{2}x_0 - 4)^2} = \frac{\left|3x_0 + 4(\tfrac{1}{2}x_0 + 4) - 11\right|}{\sqrt{9 + 16}}$$

$$\sqrt{64 - 16x_0 + x_0^2 + (-1 - \tfrac{1}{2}x_0)^2} = \frac{|3x_0 + 2x_0 + 16 - 11|}{5}$$

$$\sqrt{64 - 16x_0 + x_0^2 + 1 + x_0 + \tfrac{1}{4}x_0^2} = \frac{|5x_0 + 5|}{5}$$

$$\sqrt{\tfrac{5}{4}x_0^2 - 15x_0 + 65} = \frac{5|x_0 + 1|}{5} \quad /^2$$

$$\tfrac{5}{4}x_0^2 - 15x_0 + 65 = (x_0 + 1)^2$$

$$\tfrac{5}{4}x_0^2 - 15x_0 + 65 = x_0^2 + 2x_0 + 1$$

$$\tfrac{1}{4}x_0^2 - 17x_0 + 64 = 0 \quad / \cdot 4$$

$$x_0^2 - 68x_0 + 256 = 0$$

$$(x_0)_{1,2} = \frac{68 \pm \sqrt{68^2 - 4 \cdot 1 \cdot 256}}{2} = \frac{68 \pm 60}{2}$$

$$(x_0)_1 = \frac{68 - 60}{2} = 4 \qquad (x_0)_2 = \frac{68 + 60}{2} = 64$$

$$(y_0)_1 = \frac{1}{2} \cdot 4 + 4 = 6 \qquad (y_0)_2 = \frac{1}{2} \cdot 64 + 4 = 36$$

$$\implies P_1(4,6) \qquad \implies P_2(64,36)$$