Zadatak 19. Odredi jednadžbu simetrale najmanjeg kuta trokuta kojem su stranice na pravcima $3x-4y-2=0$, $4x-3y-5=0$ i $5x+12y+27=0$.

Rješenje.

$$a \;\dots\; 3x-4y-2=0$$
$$b \;\dots\; 4x-3y-5=0$$
$$c \;\dots\; 5x+12y+27=0$$

$$\{C\}=a\cap b \;\dots\; \begin{aligned} &3x-4y-2=0 \quad /\cdot(-4)\\ &4x-3y-5=0 \quad /\cdot 3\end{aligned}$$

$$\left.\begin{aligned} -12x+16y+8&=0\\ 12x-9y-15&=0\end{aligned}\right\}+$$

$$7y-7=0 \implies y=1$$
$$3x-4\cdot 1-2=0$$
$$3x=6 \implies x=2 \implies \boxed{C(2,1)}$$

$$\{B\}=a\cap c \;\dots\; \begin{aligned} &3x-4y-2=0 \quad /\cdot 3\\ &5x+12y+27=0\end{aligned}$$

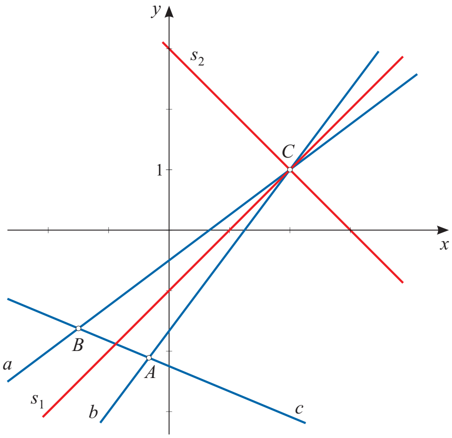
$$\left.\begin{aligned} 9x-12y-6&=0\\ 5x+12y+27&=0\end{aligned}\right\}+$$

$$14x+21=0 \implies x=-\frac{3}{2}$$
$$3\cdot\left(-\frac{3}{2}\right)-4y-2=0$$
$$-4y=2+\frac{9}{2} \implies y=-\frac{13}{8} \implies \boxed{B\left(-\frac{3}{2},-\frac{13}{8}\right)}$$

$$\{A\}=b\cap c \;\dots\; \begin{aligned} &4x-3y-5=0 \quad /\cdot 4\\ &5x+12y+27=0\end{aligned}$$

$$\left.\begin{aligned} 16x-12y-20&=0\\ 5x+12y+27&=0\end{aligned}\right\}+$$

$$21x+7=0 \implies x=-\frac{1}{3}$$
$$4\cdot\left(-\frac{1}{3}\right)-3y-5=0$$
$$-3y=\frac{19}{3} \implies y=-\frac{19}{9} \implies \boxed{A\left(-\frac{3}{2},-\frac{13}{8}\right)}$$



Iz slike se vidi da je najmanji kut γ. Tražimo simetralu s_γ. Za točku $T \in s_\gamma$ vrijedi:

$$\frac{|3x-4y-2|}{\sqrt{9+16}} = \frac{|4x-3y-5|}{\sqrt{16+9}}$$

$$\frac{|3x-4y-2|}{5} = \frac{|4x-3y-5|}{5} \quad \Big/ \cdot 5$$

$$|3x-4y-2| = |4x-3y-5|$$

1) $3x-4y-2=-4x+3y+5$

$7x-7y-7=0 \quad \Big/ :7$

$x-y-1=0 \quad \ldots \quad s_1$

2) $3x-4y-2=4x-3y-5$

$x+y-3=0 \quad \ldots \quad s_2$

Dobili smo dva pravca. Jedan od njih je simetrala tupog kuta, a drugi šiljastog kuta pri vrhu C. Pogledajmo sliku.

Traženo rješenje je $\boxed{x-y-1=0}$.