

Zadatak 15. Riješi na intervalu $[0, 2\pi]$ nejednadžbu

$$\sin x + \sqrt{3} \cos x > 0.$$

Rješenje.

I. kvadrant $\sin x > 0, \quad \cos x > 0;$
 $\sin x + \sqrt{3} \cos x > 0$
 $\sin x > -\sqrt{3} \cos x / : \cos x$
 $\operatorname{tg} x > -\sqrt{3}, \quad \text{vrijedi } \forall x \in \left\langle 0, \frac{\pi}{2} \right\rangle$

II. kvadrant $\sin x > 0, \quad \cos x < 0;$
 $\sin x + \sqrt{3} \cos x > 0$
 $\sin x > -\sqrt{3} \cos x / : \cos x$
 $\operatorname{tg} x < -\sqrt{3} \quad \left(\operatorname{tg} \frac{2\pi}{3} = -\sqrt{3} \right)$
 $\operatorname{tg} x < \operatorname{tg} \frac{2\pi}{3} \implies x \in \left[\frac{\pi}{2}, \frac{2\pi}{3} \right)$

III. kvadrant $\sin x < 0, \quad \cos x < 0;$
 $\sin x + \sqrt{3} \cos x > 0$
 $\sin x > -\sqrt{3} \cos x / : \cos x$
 $\operatorname{tg} x < -\sqrt{3} \implies \text{nema rješenja jer je u III. kvadrantu } \operatorname{tg} x > 0, \forall x$

IV. kvadrant $\sin x < 0, \quad \cos x > 0;$
 $\sin x + \sqrt{3} \cos x > 0$
 $\sin x > -\sqrt{3} \cos x / : \cos x$
 $\operatorname{tg} x > -\sqrt{3} \quad \left(\operatorname{tg} \frac{5\pi}{3} = -\sqrt{3} \right)$
 $\operatorname{tg} x > \operatorname{tg} \frac{5\pi}{3} \implies x \in \left\langle \frac{5\pi}{3}, 2\pi \right\rangle$

Unija rješenja: $\left\langle 0, \frac{2\pi}{3} \right\rangle \cup \left\langle \frac{5\pi}{3}, 2\pi \right\rangle.$