

Zadatak 2. Pojednostavni:

- 1) $\frac{1 - \sin^2 x}{\cos^2 x - 1}$;
- 2) $(1 + \operatorname{tg}^2 x) \cdot \cos^2 x$;
- 3) $\frac{\cos^2 x}{1 - \cos^2 x} \cdot \operatorname{tg}^2 x$;
- 4) $\frac{\sin^3 x + \cos^3 x}{1 - \sin x \cdot \cos x} = \sin x + \cos x$;
- 5) $\frac{1 + \operatorname{tg} x + \operatorname{tg}^2 x}{1 + \operatorname{ctg} x + \operatorname{ctg}^2 x} = \operatorname{tg}^2 x$;
- 6) $\frac{\operatorname{tg} x + \operatorname{ctg} x - 2}{\operatorname{tg} x + \operatorname{ctg} x + 2} = \left(\frac{\operatorname{tg} x - 1}{\operatorname{tg} x + 1}\right)^2$;
- 7) $(1 + \operatorname{ctg}^2 x) \cdot \sin^2 x$;
- 8) $\frac{1 - \sin^4 x - \cos^4 x}{\cos^4 x} = 2 \operatorname{tg}^2 x$;
- 9) $\frac{\sin^2 x}{\sin^2 x - 1} \cdot \operatorname{ctg}^2 x$.

Rješenje.

- 1)
$$\frac{1 - \sin^2 x}{\cos^2 x - 1} = \frac{\sin^2 x + \cos^2 x - \sin^2 x}{\cos^2 x - (\sin^2 x + \cos^2 x)} = \frac{\cos^2 x}{\cos^2 x - \sin^2 x - \cos^2 x} =$$

$$\frac{\cos^2 x}{-\sin^2 x} = -\frac{\cos^2 x}{\sin^2 x} = -\operatorname{ctg}^2 x;$$
- 2)
$$(1 + \operatorname{tg}^2 x) \cdot \cos^2 x = \left(1 + \frac{\sin^2 x}{\cos^2 x}\right) \cdot \cos^2 x = \frac{\cos^2 x + \sin^2 x}{\cos^2 x} \cdot \cos^2 x =$$

$$\frac{1}{\cos^2 x} \cdot \cos^2 x = 1;$$
- 3)
$$\frac{\cos^2 x}{1 - \cos^2 x} \cdot \operatorname{tg}^2 x = \frac{\cos^2 x}{\sin^2 x + \cos^2 x - \cos^2 x} \cdot \operatorname{tg}^2 x = \frac{\cos^2 x}{\sin^2 x} \cdot \frac{\sin^2 x}{\cos^2 x} = 1;$$
- 4)
$$\frac{\sin^3 x + \cos^3 x}{1 - \sin x \cdot \cos x} = \frac{(\sin x + \cos x)(\sin^2 x - \sin x \cos x + \cos^2 x)}{1 - \sin x \cdot \cos x} =$$

$$\frac{(\sin x + \cos x)(1 - \sin x \cdot \cos x)}{1 - \sin x \cdot \cos x} = \sin x + \cos x;$$
- 5)
$$\frac{1 + \operatorname{tg} x + \operatorname{tg}^2 x}{1 + \operatorname{ctg} x + \operatorname{ctg}^2 x} = \frac{1 + \operatorname{tg} x + \operatorname{tg}^2 x}{1 + \frac{1}{\operatorname{tg} x} + \frac{1}{\operatorname{tg}^2 x}} = \frac{1 + \operatorname{tg} x + \operatorname{tg}^2 x}{\frac{\operatorname{tg}^2 x + \operatorname{tg} x + 1}{\operatorname{tg}^2 x}} = \operatorname{tg}^2 x;$$
- 6)
$$\frac{\operatorname{tg} x + \operatorname{ctg} x - 2}{\operatorname{tg} x + \operatorname{ctg} x + 2} = \frac{\operatorname{tg} x + \frac{1}{\operatorname{tg} x} - 2}{\operatorname{tg} x + \frac{1}{\operatorname{tg} x} + 2} = \frac{\frac{\operatorname{tg}^2 x + 1 - 2 \operatorname{tg} x}{\operatorname{tg} x}}{\frac{\operatorname{tg}^2 x + 1 + 2 \operatorname{tg} x}{\operatorname{tg} x}} = \frac{(\operatorname{tg} x - 1)^2}{(\operatorname{tg} x + 1)^2} =$$

$$\left(\frac{\operatorname{tg} x - 1}{\operatorname{tg} x + 1}\right)^2;$$
- 7)
$$(1 + \operatorname{ctg}^2 x) \cdot \sin^2 x = \left(1 + \frac{\cos^2 x}{\sin^2 x}\right) \cdot \sin^2 x = \frac{\sin^2 x + \cos^2 x}{\sin^2 x} \cdot \sin^2 x = 1;$$

$$\begin{aligned} 8) \frac{1 - \sin^4 x - \cos^4 x}{\cos^2 x(1 + \sin^2 x - \cos^2 x)} &= \frac{(1 - \sin^2 x)(1 + \sin^2 x) - \cos^4 x}{\cos^4 x} = \frac{\cos^2 x \cdot (1 + \sin^2 x) - \cos^4 x}{\cos^4 x} = \\ &= \frac{\cos^4 x}{\cos^4 x} = \frac{\sin^2 x + \sin^2 x}{\cos^2 x} = \frac{2 \cdot \sin^2 x}{\cos^2 x} = 2 \operatorname{tg}^2 x; \\ 9) \frac{\sin^2 x}{\sin^2 x - 1} \cdot \operatorname{ctg}^2 x &= \frac{\sin^2 x}{\sin^2 x - (\sin^2 x + \cos^2 x)} \cdot \frac{\cos^2 x}{\sin^2 x} = \frac{\sin^2 x}{\sin^2 x - \sin^2 x - \cos^2 x} \cdot \\ \frac{\cos^2 x}{\sin^2 x} &= \frac{\sin^2 x}{-\cos^2 x} \cdot \frac{\cos^2 x}{\sin^2 x} = -1. \end{aligned}$$