

Zadatak 5. 1) Ako je $x \in \langle \frac{\pi}{2}, \pi \rangle$, onda je $\sqrt{\frac{1+\sin x}{1-\sin x}} - \sqrt{\frac{1-\sin x}{1+\sin x}} = -2 \operatorname{tg} x$.

2) Ako je $x \in \langle \pi, \frac{3\pi}{2} \rangle$, onda je $\sqrt{\frac{1-\cos x}{1+\cos x}} - \sqrt{\frac{1+\cos x}{1-\cos x}} = 2 \operatorname{ctg} x$.

Rješenje.

$$1) \sqrt{\frac{1+\sin x}{1-\sin x}} - \sqrt{\frac{1-\sin x}{1+\sin x}} = \frac{\sqrt{1+\sin x}}{\sqrt{1-\sin x}} - \frac{\sqrt{1-\sin x}}{\sqrt{1+\sin x}} = \frac{\sqrt{1+\sin x}^2 - \sqrt{1-\sin x}^2}{\sqrt{(1-\sin x)(1+\sin x)}} =$$

$$\frac{2 \sin x}{\sqrt{1-\sin^2 x}} = \frac{2 \sin x}{\sqrt{\cos^2 x}} = \left(x \in \langle \frac{\pi}{2}, \pi \rangle, \cos x < 0 \right) = \frac{2 \sin x}{-\cos x} =$$

$$-2 \cdot \frac{\sin x}{\cos x} = -2 \operatorname{tg} x;$$

$$2) \sqrt{\frac{1-\cos x}{1+\cos x}} - \sqrt{\frac{1+\cos x}{1-\cos x}} = \frac{\sqrt{1-\cos x}^2 - \sqrt{1+\cos x}^2}{\sqrt{(1-\cos x)(1+\cos x)}} = \frac{1-\cos x - (1+\cos x)}{\sqrt{1-\cos^2 x}} =$$

$$\frac{-2 \cos x}{\sqrt{\sin^2 x}} = \left(x \in \langle \pi, \frac{3\pi}{2} \rangle, \sin x < 0 \right) = \frac{-2 \cos x}{-\sin x} = 2 \cdot \frac{\cos x}{\sin x} = 2 \operatorname{ctg} x.$$