Zadatak 6. Dokaži da za sve x, $x \in \langle 0, \frac{\pi}{2} \rangle$ vrijedi $\operatorname{tg} x + \operatorname{ctg} x \geqslant 2$.

Rješenje. $x \in \langle 0, \frac{\pi}{2} \rangle$

$$\operatorname{tg} x + \operatorname{ctg} x = \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} = \frac{\sin^2 x + \cos^2 x}{\sin x \cdot \cos x} = \frac{1}{\sin x \cdot \cos x}$$

Zbog $0 < \sin x < 1$, $0 < \cos x < 1$ sljedi da je $< 0 \sin x \cdot \cos x < 1$. Produkt $\sin x \cdot \cos x$ maksimalnu vrijednost poprima za $x = \dfrac{\pi}{4}$ i ona iznosi:

$$\sin \frac{\pi}{4} \cdot \cos \frac{\pi}{4} = \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2}}{2} = \frac{1}{2},$$

dakle

$$\operatorname{tg} x + \operatorname{ctg} x \geqslant \frac{1}{\sin \frac{\pi}{4} \cdot \cos \frac{\pi}{4}} = \frac{1}{\frac{1}{2}} = 2.$$