

Zadatak 21. Iz dane vrijednosti jedne izračunaj vrijednosti ostalih trigonometrijskih funkcija:

$$1) \sin x = \frac{15}{17}, \frac{\pi}{2} < x < \pi;$$

$$2) \cos x = -\frac{5}{13}, \pi < x < \frac{3\pi}{2};$$

$$3) \operatorname{tg} x = -3\frac{3}{7}, -\frac{7\pi}{2} < x < -3\pi;$$

$$4) \operatorname{ctg} x = \frac{15}{8}, -5\pi < x < -\frac{9\pi}{2}.$$

Rješenje. 1) $\sin x = \frac{15}{17}, \frac{\pi}{2} < x < \pi$ (II. kvadrant: $\sin x > 0, \cos x < 0,$
 $\operatorname{tg} x, \operatorname{ctg} x < 0$)

$$\cos x = -\sqrt{1 - \sin^2 x} = -\sqrt{1 - \left(\frac{15}{17}\right)^2} = -\sqrt{\frac{64}{289}} = -\frac{8}{17},$$

$$\operatorname{tg} x = \frac{\sin x}{\cos x} = -\frac{15}{8}$$

$$\operatorname{ctg} x = \frac{\cos x}{\sin x} = -\frac{8}{15}$$

2) $\cos x = -\frac{5}{13}, \pi < x < \frac{3\pi}{2}$ (III. kvadrant: $\sin x > 0, \cos x < 0,$
 $\operatorname{tg} x, \operatorname{ctg} x > 0$)

$$\sin x = -\sqrt{1 - \cos^2 x} = -\sqrt{1 - \left(-\frac{5}{13}\right)^2} = -\sqrt{\frac{25}{169}} = -\frac{12}{13},$$

$$\operatorname{tg} x = \frac{\sin x}{\cos x} = \frac{12}{5}$$

$$\operatorname{ctg} x = \frac{\cos x}{\sin x} = \frac{5}{12}$$

3) $\operatorname{tg} x = -3\frac{3}{7}, -\frac{7\pi}{2} < x < -3\pi$, tj. vrijedi $-\frac{7\pi}{2} + 4\pi < x < -3\pi + 4\pi$,
odnosno $\frac{\pi}{2} < x < \pi$ (II. kvadrant: $\sin x > 0, \cos x < 0, \operatorname{tg} x, \operatorname{ctg} x < 0$)

$$\operatorname{ctg} x = \frac{1}{\operatorname{tg} x} = -\frac{7}{24}$$

$$\cos x = -\sqrt{\frac{1}{1 + \operatorname{tg}^2 x}} = -\sqrt{\frac{1}{1 + \frac{576}{49}}} = -\sqrt{\frac{1}{\frac{625}{49}}} = -\sqrt{\frac{49}{625}} = -\frac{7}{25}$$

$$\sin x = \sqrt{1 - \cos^2 x} = \sqrt{1 - \left(-\frac{7}{25}\right)^2} = \sqrt{1 - \frac{49}{625}} = \sqrt{\frac{576}{625}} = \frac{24}{25}$$

4) $\operatorname{ctg} x = \frac{15}{8}, -5\pi < x < -\frac{9\pi}{2}$ tj. vrijedi $-5\pi + 6\pi < x < -\frac{9\pi}{2} + 6\pi$,

odnosno $\pi < x < \frac{3\pi}{2}$ (III. kvadrant: $\sin x > 0$, $\cos x < 0$, $\operatorname{tg} x$, $\operatorname{ctg} x > 0$)

$$\operatorname{tg} x = \frac{1}{\operatorname{ctg} x} = \frac{8}{15}$$

$$\cos x = -\sqrt{\frac{1}{1 + \operatorname{tg}^2 x}} = -\sqrt{\frac{1}{1 + \frac{64}{225}}} = -\sqrt{\frac{1}{\frac{289}{225}}} = -\sqrt{\frac{225}{289}} = -\frac{15}{17}$$

$$\sin x = -\sqrt{1 - \cos^2 x} = -\sqrt{1 - \left(\frac{8}{15}\right)^2} = -\sqrt{1 - \frac{64}{225}} = -\sqrt{\frac{161}{225}} = -\frac{8}{15}$$