

Zadatak 3. Za sve realne brojeve x vrijedi

$$\left| \frac{3 \sin x}{2 + \cos x} \right| \leq \sqrt{3}. \text{ Dokaži!}$$

Rješenje.

$$\left| \frac{3 \sin x}{2 + \cos x} \right| \leq \sqrt{3} \quad / \cdot |2 + \cos x|$$

$$|3 \sin x| \leq \sqrt{3} \underbrace{|2 + \cos x|}_{>0}$$

$$|3 \sin x| \leq \sqrt{3}(2 + \cos x) / ^2$$

$$9 \sin^2 x \leq 3(2 + \cos x)^2$$

$$9 \sin^2 x \leq 3(4 + 4 \cos x + \cos^2 x)$$

$$9(1 - \cos^2 x) \leq 12 + 12 \cos x + 3 \cos^2 x$$

$$9 - 9 \cos^2 x \leq 12 + 12 \cos x + 3 \cos^2 x$$

$$-12 \cos^2 x - 12 \cos x - 3 \leq 0 \quad / \cdot (-1)$$

$$3(4 \cos^2 x + 4 \cos x + 1) \geq 0$$

$$3(2 \cos x + 1)^2 \geq 0$$