Zadatak 3.

1) $\operatorname{tg}\left(\frac{\pi}{3}+x\right)+\operatorname{ctg}\left(\frac{\pi}{6}-x\right)=\frac{2}{\sqrt{3}}$;

2) $\cos\left(\frac{\pi}{4}+x\right)\cdot\sin\left(\frac{\pi}{4}-x\right)=\frac{3}{4}$;

3) $\sin\left(x+\frac{\pi}{6}\right)+\cos\left(\frac{\pi}{3}-x\right)=1$;

4) $\operatorname{tg}\left(\frac{5\pi}{6}-x\right)\cdot\operatorname{ctg}\left(x-\frac{\pi}{3}\right)=3$.

Rješenje.

1)

$$\operatorname{tg}\left(\frac{\pi}{3}+x\right)+\operatorname{ctg}\left(\frac{\pi}{6}-x\right)=\frac{2}{\sqrt{3}}$$

$$\frac{\operatorname{tg}\frac{\pi}{3}+\operatorname{tg}x}{1-\operatorname{tg}\frac{\pi}{3}\operatorname{tg}x}+\frac{\operatorname{ctg}\frac{\pi}{6}\cdot\operatorname{ctg}x+1}{\operatorname{ctg}x-\operatorname{ctg}\frac{\pi}{6}}=\frac{2}{\sqrt{3}}$$

$$\frac{\sqrt{3}+\operatorname{tg}x}{1-\sqrt{3}\operatorname{tg}x}+\frac{\sqrt{3}\operatorname{ctg}x+1}{\operatorname{ctg}x-\sqrt{3}}=\frac{2}{\sqrt{3}}$$

$$\frac{\sqrt{3}+\operatorname{tg}x}{1-\sqrt{3}\operatorname{tg}x}+\frac{\frac{\sqrt{3}}{\operatorname{tg}x}+1}{\frac{1}{\operatorname{tg}x}-\sqrt{3}}=\frac{2}{\sqrt{3}}$$

$$\frac{\sqrt{3}+\operatorname{tg}x}{1-\sqrt{3}\operatorname{tg}x}+\frac{\frac{\sqrt{3}+\operatorname{tg}x}{\operatorname{tg}x}}{\frac{1-\sqrt{3}\operatorname{tg}x}{\operatorname{tg}x}}=\frac{2}{\sqrt{3}}\quad\Big/\cdot\sqrt{3}(1-\sqrt{3}\operatorname{tg}x)$$

$$\sqrt{3}(\sqrt{3}+\operatorname{tg}x+\sqrt{3}+\operatorname{tg}x)=2(1-\sqrt{3}\operatorname{tg}x)$$

$$\sqrt{3}(2\sqrt{3}+2\operatorname{tg}x)=2(1-\sqrt{3}\operatorname{tg}x)$$

$$2\sqrt{3}(\sqrt{3}+\operatorname{tg}x)=2(1-\sqrt{3}\operatorname{tg}x)\quad /:2$$

$$3+\sqrt{3}\operatorname{tg}x=1-\sqrt{3}\operatorname{tg}x$$

$$2\sqrt{3}\operatorname{tg}x=-2\quad /:2\sqrt{3}$$

$$\operatorname{tg}x=-\frac{\sqrt{3}}{3}$$

$$x=\frac{5\pi}{6}+k\cdot\pi,\quad k\in\mathbf{Z};$$

2)

$$\cos\left(\frac{\pi}{4}+x\right)\cdot\sin\left(\frac{\pi}{4}-x\right)=\frac{3}{4}$$

$$\left(\cos\frac{\pi}{4}\cos x-\sin\frac{\pi}{4}\sin x\right)\cdot\left(\sin\frac{\pi}{4}\cos x-\cos\frac{\pi}{4}\sin x\right)=\frac{3}{4}$$

$$\left(\frac{\sqrt{2}}{2}\cos x-\frac{\sqrt{2}}{2}\sin x\right)\cdot\left(\frac{\sqrt{2}}{2}\cos x-\frac{\sqrt{2}}{2}\sin x\right)=\frac{3}{4}$$

$$\frac{\sqrt{2}}{2}(\cos x-\sin x)\cdot\frac{\sqrt{2}}{2}(\cos x-\sin x)=\frac{3}{4}$$

$$\frac{2}{4}(\cos x - \sin x)^2 = \frac{3}{4}$$
$$(\cos x - \sin x)^2 = \frac{3}{2}$$
$$\cos^2 x - 2\cos x \sin x + \sin^2 x = \frac{3}{2}$$
$$-2\cos x \sin x = \frac{1}{2}$$
$$2\cos x \sin x = -\frac{1}{2}$$
$$\sin 2x = -\frac{1}{2}$$
$$2x_1 = \frac{5\pi}{6} + 2k\pi \qquad 2x_2 = \frac{7\pi}{6} + 2k\pi$$
$$x_1 = \frac{5\pi}{12} + k\pi \qquad x_2 = \frac{7\pi}{12} + k\pi, \quad k \in \mathbf{Z};$$

3)

$$\sin\left(x + \frac{\pi}{6}\right) + \cos\left(\frac{\pi}{3} - x\right) = 1$$
$$\sin x \cos\frac{\pi}{6} + \cos x \sin\frac{\pi}{6} + \cos\frac{\pi}{3}\cos x + \sin\frac{\pi}{3}\sin x = 1$$
$$\frac{\sqrt{3}}{2}\sin x + \frac{1}{2}\cos x + \frac{1}{2}\cos x + \frac{\sqrt{3}}{2}\sin x = 1$$
$$\sqrt{3}\sin x + \cos x = 1$$
$$2\left(\frac{\sqrt{3}}{2}\sin x + \frac{1}{2}\cos x\right) = 1$$
$$\sin\frac{\pi}{3}\sin x + \cos\frac{\pi}{3}\cos x = \frac{1}{2}$$
$$\cos\left(\frac{\pi}{3} - x\right) = \frac{1}{2}$$
$$\frac{\pi}{3} - x_1 = \frac{\pi}{3} + 2k\pi \qquad \frac{\pi}{3} - x_2 = \frac{5\pi}{3} + 2k\pi$$
$$-x_1 = 2k\pi \qquad -x_2 = \frac{4\pi}{3} + 2k\pi$$
$$x_1 = 2k\pi \qquad x_2 = -\frac{4\pi}{3} + 2k\pi$$
$$x_2 = \frac{2\pi}{3} + 2k\pi$$

4)

$$\operatorname{tg}\left(\frac{5\pi}{6}-x\right)\cdot\operatorname{ctg}\left(x-\frac{\pi}{3}\right)=3$$

$$\frac{\operatorname{tg}\left(\frac{5\pi}{6}-x\right)}{\operatorname{tg}\left(x-\frac{\pi}{3}\right)}=3$$

$$\frac{\dfrac{\operatorname{tg}\frac{5\pi}{6}-\operatorname{tg}x}{1+\operatorname{tg}\frac{5\pi}{6}\cdot\operatorname{tg}x}}{\dfrac{\operatorname{tg}x-\operatorname{tg}\frac{\pi}{3}}{1+\operatorname{tg}x\operatorname{tg}\frac{\pi}{3}}}=3$$

$$\frac{\dfrac{-\frac{\sqrt{3}}{3}-\operatorname{tg}x}{1-\frac{\sqrt{3}}{3}\operatorname{tg}x}}{\dfrac{\operatorname{tg}x-\sqrt{3}}{1+\sqrt{3}\operatorname{tg}x}}=3$$

$$\frac{\left(-\frac{\sqrt{3}}{3}-\operatorname{tg}x\right)(1+\sqrt{3}\operatorname{tg}x)}{\left(1-\frac{\sqrt{3}}{3}\operatorname{tg}x\right)(\operatorname{tg}x-\sqrt{3})}=3$$

$$\frac{-\frac{\sqrt{3}}{3}(1+\sqrt{3}\operatorname{tg}x)(1+\sqrt{3}\operatorname{tg}x)}{-\frac{\sqrt{3}}{3}(\operatorname{tg}x-\sqrt{3})(\operatorname{tg}x-\sqrt{3})}=3$$

$$\frac{(1+\sqrt{3}\operatorname{tg}x)^2}{(\operatorname{tg}x-\sqrt{3})^2}=3$$

$$(1+\sqrt{3}\operatorname{tg}x)^2=3(\operatorname{tg}x-\sqrt{3})^2$$

$$1+2\sqrt{3}\operatorname{tg}x+3\operatorname{tg}^2x=3\operatorname{tg}^2x-6\sqrt{3}\operatorname{tg}x+9$$

$$8\sqrt{3}\operatorname{tg}x=8$$

$$\operatorname{tg}x=\frac{\sqrt{3}}{3}$$

$$x=\frac{\pi}{6}+k\cdot\pi,\quad k\in\mathbf{Z}.$$