Zadatak 5.
1) $\sin x\cdot\cos 2x=0$;
2) $\sin^2 x=2\sin x$;
3) $\sin^3 2x-\sin 2x=0$;
4) $\cos^4 x-\cos^2 x=0$;
5) $\sin^2\left(\frac{\pi}{4}-x\right)+\cos\left(\frac{\pi}{4}+x\right)=0$;
6) $\operatorname{tg}^2\left(\frac{\pi}{3}+x\right)=\operatorname{ctg}\left(\frac{\pi}{6}-x\right)$.

Rješenje. **1)**

$$\sin x\cdot\cos 2x=0 \implies \sin x_1=0 \quad \text{ili} \quad \cos 2x_2=0$$
$$x_1=k\pi \qquad 2x_2=\frac{(2k+1)\pi}{2}$$
$$x_2=\frac{(2k+1)\pi}{4} \qquad k\in\mathbf{Z};$$

2)

$$\sin^2 x=2\sin x$$
$$\sin^2 x-2\sin x=0$$
$$\sin x(\sin x-2)=0$$
$$|\sin x|\leqslant 1 \implies \sin x=0,\quad x=k\pi,\quad k\in\mathbf{Z};$$

3)

$$\sin^3 2x-\sin 2x=0$$
$$\sin 2x(\sin^2 2x-1)=0 \implies \sin 2x_1=0 \quad \text{ili} \quad \sin^2 2x_2=1/\sqrt{\ }$$
$$2x_1=k\pi \qquad \sin 2x_2=\pm 1$$
$$x_1=\frac{k\pi}{2} \qquad 2x_2=\frac{\pi}{2}+k\pi$$
$$x_1=\frac{2k\pi}{4} \qquad x_2=\frac{(1+2k)\pi}{4}$$
$$x_1\cup x_2 \implies \quad x=\frac{k\pi}{4} \qquad k\in\mathbf{Z};$$

4)

$$\cos^4 x-\cos^2 x=0$$
$$\cos^2 x(\cos^2 x-1)=0 \implies \cos^2 x_1=0 \quad \text{ili} \quad \cos^2 x_2=1$$
$$\cos x_1=0 \qquad \cos x_2=\pm 1$$
$$x_1=\frac{\pi}{2}+k\pi \qquad x_2=k\pi$$
$$x_1=\frac{(1+2k)\pi}{2} \qquad x_2=\frac{2k\pi}{2}$$
$$x_1\cup x_2 \implies \quad x=\frac{k\pi}{2}, \qquad k\in\mathbf{Z};$$

5)

$$\sin^2\left(\frac{\pi}{4}-x\right)+\cos\left(\frac{\pi}{4}+x\right)=0$$
$$\left(\sin\frac{\pi}{4}\cos x-\cos\frac{\pi}{4}\sin x\right)^2+\cos\frac{\pi}{4}\cos x-\sin\frac{\pi}{4}\sin x=0$$
$$\left(\frac{\sqrt{2}}{2}\cos x-\frac{\sqrt{2}}{2}\sin x\right)^2+\frac{\sqrt{2}}{2}\cos x-\frac{\sqrt{2}}{2}\sin x=0$$
$$\left(\frac{\sqrt{2}}{2}\cos x-\frac{\sqrt{2}}{2}\sin x\right)\left(\frac{\sqrt{2}}{2}\cos x-\frac{\sqrt{2}}{2}\sin x+1\right)=0$$

1°
$$\frac{\sqrt{2}}{2}\cos x-\frac{\sqrt{2}}{2}\sin x=0$$
$$\cos x-\sin x=0 \quad / : \cos x$$
$$1-\operatorname{tg} x=0$$
$$\operatorname{tg} x=1$$
$$x_1=\frac{\pi}{4}+k\pi,\ k\in\mathbf{Z}$$

2°
$$\frac{\sqrt{2}}{2}\cos x-\frac{\sqrt{2}}{2}\sin x+1=0$$
$$\frac{\sqrt{2}}{2}\cos x-\frac{\sqrt{2}}{2}\sin x=-1$$
$$\cos\frac{\pi}{4}\cos x-\sin\frac{\pi}{4}\sin x=-1$$
$$\cos\left(\frac{\pi}{4}+x\right)=-1$$
$$\frac{\pi}{4}+x=\pi+2k\pi$$
$$x_2=\frac{3\pi}{4}+2k\pi$$

6)

$$\operatorname{tg}^2\left(\frac{\pi}{3}+x\right)=\operatorname{ctg}\left(\frac{\pi}{6}-x\right)$$
$$\left(\frac{\operatorname{tg}\frac{\pi}{3}+\operatorname{tg} x}{1-\operatorname{tg}\frac{\pi}{3}\operatorname{tg} x}\right)^2=\frac{1+\operatorname{tg}\frac{\pi}{6}\operatorname{tg} x}{\operatorname{tg}\frac{\pi}{6}-\operatorname{tg} x}$$
$$\left(\frac{\sqrt{3}+\operatorname{tg} x}{1-\sqrt{3}\operatorname{tg} x}\right)^2=\frac{1+\frac{\sqrt{3}}{3}\operatorname{tg} x}{\frac{\sqrt{3}}{3}-\operatorname{tg} x}$$
$$\frac{(\sqrt{3}+\operatorname{tg} x)^2}{(1-\sqrt{3}\operatorname{tg} x)^2}=\frac{\frac{\sqrt{3}}{3}(\sqrt{3}+\operatorname{tg} x)}{\frac{\sqrt{3}}{3}(1-\sqrt{3}\operatorname{tg} x)}$$
$$\frac{(\sqrt{3}+\operatorname{tg} x)^2}{(1-\sqrt{3}\operatorname{tg} x)^2}-\frac{\sqrt{3}+\operatorname{tg} x}{1-\sqrt{3}\operatorname{tg} x}=0$$
$$\frac{\sqrt{3}+\operatorname{tg} x}{1-\sqrt{3}\operatorname{tg} x}\left(\frac{\sqrt{3}+\operatorname{tg} x}{1-\sqrt{3}\operatorname{tg} x}-1\right)=0$$

$$\frac{\sqrt{3}+\operatorname{tg} x}{1-\sqrt{3}\operatorname{tg} x}=0 \implies \sqrt{3}+\operatorname{tg} x=0$$

$$\operatorname{tg} x=-\sqrt{3}$$

$$x_1=\frac{2\pi}{3}+k\pi, \quad k\in\mathbf{Z}$$

$$\frac{\sqrt{3}+\operatorname{tg} x}{1-\sqrt{3}\operatorname{tg} x}-1=0 \implies \sqrt{3}+\operatorname{tg} x-1+\sqrt{3}\operatorname{tg} x=0$$

$$\sqrt{3}-1+\operatorname{tg} x(1+\sqrt{3})=0$$

$$\operatorname{tg} x(1+\sqrt{3})=1-\sqrt{3}$$

$$\operatorname{tg} x=\frac{1-\sqrt{3}}{1+\sqrt{3}}$$

$$\operatorname{tg} x=\frac{\operatorname{tg}\frac{\pi}{4}-\operatorname{tg}\frac{\pi}{3}}{1+\operatorname{tg}\frac{\pi}{4}\cdot\operatorname{tg}\frac{\pi}{3}}$$

$$\operatorname{tg} x=\operatorname{tg}\left(\frac{\pi}{4}-\frac{\pi}{3}\right)$$

$$\operatorname{tg} x=\operatorname{tg}\left(-\frac{\pi}{12}\right)$$

$$x_2=-\frac{\pi}{12}+k\pi, \quad k\in\mathbf{Z}$$