Zadatak 6.

1) $\sin x - \sqrt{3}\cos x = 0\,;$

2) $\sqrt{3}\sin x + \cos x = 0\,;$

3) $\sin 2x + \sqrt{3}\cos 2x = 0\,;$

4) $\sin(\frac{\pi}{2} + x) + \operatorname{ctg}(2\pi - x) = 0\,;$

5) $\operatorname{ctg}(\frac{3\pi}{2} + x) = \cos(\frac{3\pi}{2} - x)\,.$

Rješenje. **1)**

$$
\begin{aligned}
&\sin x - \sqrt{3}\cos x = 0 \quad / : 2\\
&\frac{1}{2}\sin x - \frac{\sqrt{3}}{2}\cos x = 0\\
&\sin\frac{\pi}{6}\sin x - \cos\frac{\pi}{6}\cos x = 0\\
&-\cos\left(\frac{\pi}{6} + x\right) = 0\\
&\cos\left(\frac{\pi}{6} + x\right) = 0\\
&\frac{\pi}{6} + x = \frac{\pi}{2} + k\pi\\
&x = \frac{\pi}{3} + k\pi, \quad k \in \mathbf{Z};
\end{aligned}
$$

2)

$$
\begin{aligned}
&\sqrt{3}\sin x + \cos x = 0 \quad / : 2\\
&\frac{\sqrt{3}}{2}\sin x + \frac{1}{2}\cos x = 0\\
&\sin\frac{\pi}{3}\sin x + \cos\frac{\pi}{3}\cos x = 0\\
&\cos\left(\frac{\pi}{3} - x\right) = 0\\
&\frac{\pi}{3} - x = -\frac{\pi}{2} - k\pi\\
&-x = -\frac{5\pi}{6} - k\pi\\
&x = \frac{5\pi}{6} + k\pi, \quad k \in \mathbf{Z};
\end{aligned}
$$

3)

$$\sin 2x + \sqrt{3}\cos 2x = 0 \quad / : 2$$
$$\frac{1}{2}\sin 2x + \frac{\sqrt{3}}{2}\cos 2x = 0$$
$$\cos\frac{\pi}{3}\sin 2x + \sin\frac{\pi}{3}\cos 2x = 0$$
$$\sin\left(2x + \frac{\pi}{3}\right) = 0$$
$$2x + \frac{\pi}{3} = k\pi$$
$$2x = -\frac{\pi}{3} + k\pi$$
$$x = -\frac{\pi}{6} + k\cdot\frac{\pi}{2}, \quad k \in \mathbf{Z}$$

4)

$$\sin\left(\frac{\pi}{2} + x\right) + \operatorname{ctg}\left(2\pi - x\right) = 0$$
$$\sin\frac{\pi}{2}\cos x + \cos\frac{\pi}{2}\sin x + \frac{1}{\operatorname{tg}\left(2\pi - x\right)} = 0$$
$$\cos x + \frac{1 + \operatorname{tg} 2\pi \operatorname{tg} x}{\operatorname{tg} 2\pi - \operatorname{tg} x} = 0$$
$$\cos x + \frac{1}{-\operatorname{tg} x} = 0$$
$$\cos x - \operatorname{ctg} x = 0$$
$$\cos x - \frac{\cos x}{\sin x} = 0$$
$$\cos x\left(1 - \frac{1}{\sin x}\right) = 0$$

1° $\cos x = 0$

$$x_1 = \frac{\pi}{2} + 2k\pi$$

2° $1 - \frac{1}{\sin x} = 0$

$$\sin x - 1 = 0$$
$$\sin x = 1$$
$$x_2 = \frac{\pi}{2} + 2k\pi, \quad k \in \mathbf{Z}$$

$$\Longrightarrow \quad x = \frac{\pi}{2} + 2k\pi, \quad k \in \mathbf{Z}$$

$x = (2k+1)\cdot\frac{\pi}{2}$, $k \in \mathbf{Z}$;

5)

$$\operatorname{ctg}\left(\frac{3\pi}{2}+x\right)=\cos\left(\frac{3\pi}{2}-x\right)$$

$$\frac{\operatorname{ctg}\frac{3\pi}{2}\operatorname{ctg}x-1}{\operatorname{ctg}\frac{3\pi}{2}+\operatorname{ctg}x}=\cos\frac{3\pi}{2}\cos x+\sin\frac{3\pi}{2}\sin x$$

$$\frac{-1}{\operatorname{ctg}x}=-\sin x$$

$$-\operatorname{tg}x+\sin x=0$$

$$-\frac{\sin x}{\cos x}+\sin x=0$$

$$\sin x\left(-\frac{1}{\cos x}+1\right)=0$$

1° $\quad \sin x=0$

$$x_1=k\pi,\quad k\in\mathbf{Z}$$

2° $\quad -\frac{1}{\cos x}+1=0$

$$-1+\cos x=0$$

$$\cos x=1$$

$$x_2=2k\pi,\quad k\in\mathbf{Z}$$

$$x_1\cup x_2\implies x=k\pi,\quad k\in\mathbf{Z}$$