

- Zadatak 7.**
- 1) $\sin^2 x - 3 \sin x + 2 = 0;$
 - 2) $4 \cos^2 x - 4 \cos x + 1 = 0;$
 - 3) $2 \sin^2 x + 5 \sin x + 2 = 0;$
 - 4) $\operatorname{tg}^2 x - (\sqrt{3} + 1) \operatorname{tg} x + \sqrt{3} = 0;$
 - 5) $\operatorname{ctg}^2 x - (\sqrt{3} - 1) \operatorname{ctg} x - \sqrt{3} = 0.$

Rješenje. 1)

$$\sin^2 x - 3 \sin x + 2 = 0$$

$$(\sin x)_{1,2} = \frac{3 \pm \sqrt{9 - 8}}{2} = \frac{3 \pm 1}{2}$$

$(\sin x)_1 = 2$ nije rješenje jer mora vrijediti $|\sin x| \leq 1$

$$(\sin x)_2 = 1 \implies x = \frac{\pi}{2} + 2k\pi, \quad k \in \mathbf{Z};$$

2)

$$4 \cos^2 x - 4 \cos x + 1 = 0$$

$$(2 \cos x - 1)^2 = 0$$

$$2 \cos x = 1$$

$$\cos x = \frac{1}{2} \implies x = \pm \frac{\pi}{3} + 2k\pi, \quad k \in \mathbf{Z};$$

3)

$$2 \sin^2 x + 5 \sin x + 2 = 0$$

$$(\sin x)_{1,2} = \frac{-5 \pm \sqrt{25 - 16}}{4} = \frac{-5 \pm 3}{4}$$

$(\sin x)_1 = -2$ nije rješenje jer mora vrijediti $|\sin x| \leq 1$

$$(\sin x)_2 = -\frac{1}{2} \implies x_1 = \frac{5\pi}{6} + 2k\pi, \quad x_2 = \frac{7\pi}{6} + 2k\pi, \quad k \in \mathbf{Z};$$

4)

$$\operatorname{tg}^2 x - (\sqrt{3} + 1) \operatorname{tg} x + \sqrt{3} = 0$$

$$(\operatorname{tg} x)_{1,2} = \frac{\sqrt{3} + 1 \pm \sqrt{(\sqrt{3} + 1)^2 - 4\sqrt{3}}}{2} = \frac{\sqrt{3} + 1 \pm \sqrt{(\sqrt{3} - 1)^2}}{2} = \frac{\sqrt{3} + 1 \pm (\sqrt{3} - 1)}{2}$$

$$(\operatorname{tg} x)_1 = \frac{\sqrt{3} + 1 - \sqrt{3} + 1}{2} = 1 \implies x_1 = \frac{\pi}{4} + k\pi, \quad k \in \mathbf{Z}$$

$$(\operatorname{tg} x)_2 = \frac{\sqrt{3} + 1 + \sqrt{3} - 1}{2} = \sqrt{3} \implies x_2 = \frac{\pi}{3} + k\pi, \quad k \in \mathbf{Z};$$

5)

$$\operatorname{ctg}^2 x - (\sqrt{3} - 1) \operatorname{ctg} x - \sqrt{3} = 0$$

$$(\operatorname{ctg} x)_{1,2} = \frac{\sqrt{3} - 1 \pm \sqrt{(\sqrt{3} - 1)^2 + 4\sqrt{3}}}{2} = \frac{\sqrt{3} - 1 \pm \sqrt{(\sqrt{3} + 1)^2}}{2} = \frac{\sqrt{3} - 1 \pm (\sqrt{3} + 1)}{2}$$

$$(\operatorname{ctg} x)_1 = \frac{\sqrt{3} - 1 - \sqrt{3} - 1}{2} = -1 \implies x_1 = -\frac{\pi}{4} + k\pi, \quad k \in \mathbf{Z}$$

$$(\operatorname{ctg} x)_2 = \frac{\sqrt{3} - 1 + \sqrt{3} + 1}{2} = \sqrt{3} \implies x_2 = \frac{\pi}{6} + k\pi, \quad k \in \mathbf{Z}.$$