Zadatak 8.

1) $2\cos^2 x - 3\sin x = 0$;
2) $\sin^2 x - \cos^2 x = \cos x$;
3) $7\sin^2 x - 5\cos^2 x + 2 = 0$;
4) $2\sin^2 3x - 5\cos 3x - 4 = 0$;
5) $\operatorname{tg} x - 4\operatorname{ctg} x = 3$;
6) $\operatorname{tg}^2 x + \operatorname{ctg}^2 x = 2$;
7) $2\cos^2 \frac{x}{3} + 3\sin\frac{x}{3} = 0$;
8) $2\sin^2 2x - \cos^2 2x = 5\cos 2x$;
9) $\sin^2\frac{x}{2} - 5\sin\frac{x}{2} = 2\cos^2\frac{x}{2}$;
10) $4\sin^4 x + 12\cos^2 x = 7$.

Rješenje. **1)**

$$
\begin{aligned}
&2\cos^2 x - 3\sin x = 0\\
&2(1-\sin^2 x) - 3\sin x = 0\\
&-2\sin^2 x - 3\sin x + 2 = 0\\
&2\sin^2 x + 3\sin x - 2 = 0\\
&(\sin x)_{1,2} = \frac{-3\pm\sqrt{9+16}}{4} = \frac{-3\pm 5}{4}\\
&(\sin x)_1 = -2 \ \text{ nije rješenje}\\
&(\sin x)_2 = \frac{1}{2} \implies x_1 = \frac{\pi}{2} + 2k\pi, \quad x_2 = \frac{5\pi}{6} + 2k\pi, \quad k\in\mathbf{Z};
\end{aligned}
$$

2)

$$
\begin{aligned}
&\sin^2 x - \cos^2 x = \cos x\\
&1-\cos^2 x - \cos^2 x - \cos x = 0\\
&-2\cos^2 x - \cos x + 1 = 0\\
&2\cos^2 x + \cos x - 1 = 0\\
&(\cos x)_{1,2} = \frac{-1\pm\sqrt{1+8}}{4} = \frac{-1\pm 3}{4}\\
&(\cos x)_1 = -1 \implies x_1 = (2k+1)\pi, \quad k\in\mathbf{Z}\\
&(\cos x)_2 = \frac{1}{2} \implies x_{2,3} = \pm\frac{\pi}{3} + 2k\pi, \quad k\in\mathbf{Z}
\end{aligned}
$$

3)

$$
\begin{aligned}
&7\sin^2 x - 5\cos^2 x + 2 = 0\\
&7\sin^2 x - 5(1-\sin^2 x) + 2 = 0\\
&7\sin^2 x - 5 + 5\sin^2 x + 2 = 0\\
&12\sin^2 x - 3 = 0\\
&\sin^2 x = \frac{1}{4}\\
&\sin x = \pm\frac{1}{2}\\
&x = \pm\frac{\pi}{6} + k\pi, \quad k \in \mathbf{Z}
\end{aligned}
$$

4)

$$
\begin{aligned}
&2\sin^2 3x - 5\cos 3x - 4 = 0\\
&2(1-\cos^2 3x) - 5\cos 3x - 4 = 0\\
&-2\cos^2 3x - 5\cos 3x - 2 = 0\\
&2\cos^2 3x + 5\cos 3x + 2 = 0\\
&(\cos 3x)_{1,2} = \frac{-5 \pm \sqrt{25-16}}{4} = \frac{-5\pm 3}{4}\\
&(\cos 3x)_1 = -2 \implies \text{nije rješenje}\\
&(\cos 3x)_2 = -\frac{1}{2} \implies 3x = \pm\frac{2\pi}{3} + 2k\pi, \quad x = \pm\frac{2\pi}{9} + \frac{2k\pi}{3}, \quad k \in \mathbf{Z}
\end{aligned}
$$

5)

$$
\begin{aligned}
&\operatorname{tg} x - 4\operatorname{ctg} x = 3\\
&\operatorname{tg} x - \frac{4}{\operatorname{tg} x} = 3 \quad / \cdot \operatorname{tg} x\\
&\operatorname{tg}^2 x - 3\operatorname{tg} x - 4 = 0\\
&(\operatorname{tg} x)_{1,2} = \frac{3 \pm \sqrt{9+16}}{2} = \frac{3 + \pm 5}{2}\\
&(\operatorname{tg} x)_1 = 4 \implies x_1 = \operatorname{arc\,tg} 4 + k\pi, \quad k \in \mathbf{Z}\\
&(\operatorname{tg} x)_2 = -1 \implies x_2 = -\frac{\pi}{4} + k\pi, \quad k \in \mathbf{Z};
\end{aligned}
$$

6)

$$
\begin{aligned}
&\operatorname{tg}^2 x + \operatorname{ctg}^2 x = 2\\
&\operatorname{tg}^2 x + \frac{1}{\operatorname{tg}^2 x} = 2 \quad \Big/ \cdot \operatorname{tg}^2 x\\
&\operatorname{tg}^4 x - 2\operatorname{tg}^2 x + 1 = 0\\
&(\operatorname{tg}^2 x)_{1,2} = \frac{2 \pm \sqrt{4-4}}{2} = 1\\
&\operatorname{tg} x = \pm 1 \implies x = \frac{\pi}{4} + k\cdot\frac{\pi}{2}, \quad k \in \mathbf{Z};
\end{aligned}
$$

7)

$$2\cos^2\frac{x}{3}+3\sin\frac{x}{3}=0$$
$$2\left(1-\sin^2\frac{x}{3}\right)+3\sin\frac{x}{3}=0$$
$$-2\sin^2\frac{x}{3}+3\sin\frac{x}{3}+2=0$$
$$2\sin^2\frac{x}{3}-3\sin\frac{x}{3}-2=0$$
$$\left(\sin\frac{x}{3}\right)_{1,2}=\frac{3\pm\sqrt{9+16}}{4}=\frac{3\pm5}{4}$$
$$\left(\sin\frac{x}{3}\right)_1=2 \implies \text{nije rješenje}$$
$$\left(\sin\frac{x}{3}\right)_2=-\frac{1}{2} \implies \frac{x_1}{3}=\frac{5\pi}{6}+2k\pi,\quad x_1=\frac{5\pi}{2}+6k\pi;$$
$$\frac{x_2}{3}=\frac{7\pi}{6}+2k\pi,\quad x_2=\frac{7\pi}{2}+6k\pi;$$

8)

$$2\sin^2 2x-\cos^2 2x=5\cos 2x$$
$$2(1-\cos^2 2x)-\cos^2 2x=5\cos 2x$$
$$-3\cos^2 2x-5\cos 2x+2=0$$
$$3\cos^2 2x+5\cos 2x-2=0$$
$$(\cos 2x)_{1,2}=\frac{-5\pm\sqrt{25+24}}{6}=\frac{-5\pm7}{6}$$
$$(\cos 2x)_1=-2 \implies \text{nije rješenje}$$
$$(\cos 2x)_2=\frac{1}{3} \implies 2x=\arccos\left(\frac{1}{3}\right)+2k\pi,\quad x=\frac{1}{2}\arccos\left(\frac{1}{3}\right)+k\pi,\quad k\in\mathbf{Z};$$

9)

$$\sin^2\frac{x}{2}-5\sin\frac{x}{2}=2\cos^2\frac{x}{2}$$
$$\sin^2\frac{x}{2}-2\left(1-\sin^2\frac{x}{2}\right)-5\sin\frac{x}{2}=0$$
$$\sin^2\frac{x}{2}-5\sin\frac{x}{2}-2=0$$
$$\left(\sin\frac{x}{2}\right)_{1,2}=\frac{5\pm\sqrt{25+24}}{6}=\frac{5\pm7}{6}$$
$$\left(\sin\frac{x}{2}\right)_1=2 \implies \text{nije rješenje}$$
$$\left(\sin\frac{x}{2}\right)_2=-\frac{1}{3} \implies x=(-1)^{k+1}\cdot 2\arcsin\frac{1}{3}+k\cdot 2\pi,\quad k\in\mathbf{Z}$$

10)

$$4\sin^4 x + 12(1-\sin^2 x) = 7$$
$$4\sin^4 x - 12\sin^2 x + 5 = 0$$
$$(\sin^2 x)_{1,2} = \frac{12 \pm \sqrt{144-80}}{8} = \frac{12 \pm 8}{8}$$
$$(\sin^2 x)_1 = \frac{5}{2} \implies \text{nije rješenje}$$
$$(\sin^2 x)_2 = \frac{1}{2}$$
$$\sin x_2 = \pm\frac{\sqrt{2}}{2} \implies x_2 = \frac{\pi}{4} + k\frac{\pi}{2} = (2k+1)\cdot\frac{\pi}{4}, \quad k \in \mathbf{Z}.$$