Zadatak 9.

1) $2\cos^4 x - 3\cos^2 x + 1 = 0$;

2) $3\operatorname{tg}^4 x - 10\operatorname{tg}^2 x + 3 = 0$;

3) $6\sin^4 x = 1 + \sin^2 x$;

4) $4\cos^4 3x + 8 = 11\sin^2 3x$;

5) $4\sin^4 \frac{x}{4} - 1 = 5\cos^2 \frac{x}{4}$.

Rješenje. **1)**

$$2\cos^4 x - 3\cos^2 x + 1 = 0$$

$$(\cos^2 x)_{1,2} = \frac{3 \pm \sqrt{9-8}}{4} = \frac{3 \pm 1}{4}$$

$$(\cos^2 x)_1 = 1 \implies \cos x_1 = \pm 1 \implies x_1 = k\pi, \quad k \in \mathbf{Z}$$

$$(\cos^2 x)_2 = \frac{1}{2} \implies \cos x_2 = \pm\frac{\sqrt{2}}{2} \implies x_2 = \frac{\pi}{4} + k \cdot \frac{\pi}{2}, \quad k \in \mathbf{Z}$$

2)

$$3\operatorname{tg}^4 x - 10\operatorname{tg}^2 x + 3 = 0$$

$$(\operatorname{tg}^2 x)_{1,2} = \frac{10 \pm \sqrt{100-36}}{6} = \frac{10 \pm 8}{6}$$

$$(\operatorname{tg}^2 x)_1 = 3 \implies \operatorname{tg} x_1 = \pm\sqrt{3} \implies x_1 = \pm\frac{\pi}{3} + k\pi, \quad k \in \mathbf{Z}$$

$$(\operatorname{tg}^2 x)_2 = \frac{1}{3} \implies \operatorname{tg} x_1 = \pm\frac{\sqrt{3}}{3} \implies x_1 = \pm\frac{\pi}{6} + k\pi, \quad k \in \mathbf{Z}$$

3)

$$6\sin^4 x = 1 + \sin^2 x$$

$$6\sin^4 x - \sin^2 x - 1 = 0$$

$$(\sin^2 x)_{1,2} = \frac{1 \pm \sqrt{1+24}}{12} = \frac{1 \pm 5}{12}$$

$$(\sin^2 x)_1 = \frac{1}{2} \implies \sin x_1 = \pm\frac{\sqrt{2}}{2} \implies x_1 = \frac{\pi}{4} + k \cdot \frac{\pi}{2}, \quad k \in \mathbf{Z}$$

$$(\sin^2 x)_2 = -\frac{1}{3} \implies \text{nema rješenja;}$$

4)

$$4\cos^4 3x + 8 = 11\sin^2 3x$$

$$4\cos^4 3x + 8 = 11(1 - \cos^2 3x)$$

$$4\cos^4 3x + 11\cos^2 3x - 3 = 0$$

$$(\cos^2 3x)_{1,2} = \frac{-11 \pm \sqrt{121+48}}{8} = \frac{-11 \pm 13}{8}$$

$$(\cos^2 3x)_1 = -3 \implies \text{nema rješenja}$$

$$(\cos^2 3x)_2 = \frac{1}{4} \implies \cos 3x = \pm\frac{1}{2} \implies 3x = \pm\frac{\pi}{3} + k\pi, \quad x = \pm\frac{\pi}{9} + k \cdot \frac{\pi}{3}, \quad k \in \mathbf{Z}$$

5)

$$4\sin^4\frac{x}{4}-1=5\cos^2\frac{x}{4}$$

$$4\sin^4\frac{x}{4}-1=5\left(1-\sin^2\frac{x}{4}\right)$$

$$4\sin^4\frac{x}{4}+5\sin^2\frac{x}{4}-6=0$$

$$\left(\sin^2\frac{x}{4}\right)_{1,2}=\frac{-5\pm\sqrt{25+96}}{8}=\frac{-5\pm 11}{8}$$

$$\left(\sin^2\frac{x}{4}\right)_1=-2 \implies \text{nije rješenje}$$

$$\left(\sin^2\frac{x}{4}\right)_2=\frac{3}{4} \implies \sin\frac{x_2}{4}=\pm\frac{\sqrt{3}}{2} \implies \frac{x_2}{4}=\pm\frac{\pi}{3}+k\pi, \quad x_2=\pm\frac{4\pi}{3}+4k\pi, \quad k\in\mathbf{Z}$$