Zadatak 10.

1) $5\sin^2 x + 4\sin\left(\frac{\pi}{2} + x\right) = 4\,;$

2) $6\cos^2 x + 5\cos\left(\frac{\pi}{2} - x\right) = 7\,;$

3) $2\cos^2\left(x + \frac{\pi}{6}\right) + 3\sin\left(\frac{\pi}{3} - x\right) + 1 = 0\,;$

4) $2\sin^2\left(\frac{\pi}{3} + x\right) - 3\cos\left(\frac{\pi}{6} - x\right) + 1 = 0\,.$

Rješenje. **1)**

$$
\begin{aligned}
&5\sin^2 x + 4\sin\left(\frac{\pi}{2} + x\right) = 4\\
&5\sin^2 x + 4\left(\sin\frac{\pi}{2}\cos x + \cos\frac{\pi}{2}\sin x\right) = 4\\
&5\sin^2 x + 4\cos x - 4 = 0\\
&5(1 - \cos^2 x) + 4\cos x - 4 = 0\\
&-5\cos^2 x + 4\cos x + 1 = 0\\
&5\cos^2 x - 4\cos x - 1 = 0\\
&(\cos x)_{1,2} = \frac{4 \pm \sqrt{16+20}}{10} = \frac{4 \pm 6}{10}\\
&(\cos x)_1 = 1 \implies x_1 = 2k\pi, \quad k \in \mathbf{Z}\\
&(\cos x)_2 = -\frac{1}{5} \implies x_2 = \pm\arccos\left(-\frac{1}{5}\right) + 2k\pi, \quad k \in \mathbf{Z}
\end{aligned}
$$

2)

$$
\begin{aligned}
&6\cos^2 x + 5\cos\left(\frac{\pi}{2} - x\right) = 7\\
&6\cos^2 x + 5\left(\cos\frac{\pi}{2}\cos x + \sin\frac{\pi}{2}\sin x\right) - 7 = 0\\
&6(1 - \sin^2 x) + 5\sin x - 7 = 0\\
&-6\sin^2 x + 5\sin x - 1 = 0\\
&6\sin^2 x - 5\sin x + 1 = 0\\
&(\sin x)_{1,2} = \frac{5 \pm \sqrt{25-24}}{12} = \frac{5 \pm 1}{12}\\
&(\sin x)_1 = \frac{1}{2} \implies x_1 = \frac{\pi}{6} + 2k\pi, \quad x_2 = \frac{5\pi}{6} + 2k\pi, \quad k \in \mathbf{Z}\\
&(\sin x)_2 = \frac{1}{3} \implies x_3 = (-1)^k \arcsin\frac{1}{3} + k\cdot\pi, \quad k \in \mathbf{Z}
\end{aligned}
$$

3)

$$2\cos^2\left(x+\frac{\pi}{6}\right)+3\sin\left(\frac{\pi}{3}-x\right)+1=0$$

$$2\left(\cos x\cos\frac{\pi}{6}-\sin x\sin\frac{\pi}{6}\right)^2+3\left(\sin\frac{\pi}{3}\cos x-\cos\frac{\pi}{3}\sin x\right)+1=0$$

$$2\left(\frac{\sqrt{3}}{2}\cos x-\frac{1}{2}\sin x\right)^2+3\left(\frac{\sqrt{3}}{2}\cos x-\frac{1}{2}\sin x\right)+1=0$$

$$2\cdot\frac{1}{4}\left(\sqrt{3}\cos x-\sin x\right)^2+\frac{3}{2}\left(\sqrt{3}\cos x-\sin x\right)+1=0 \quad /\cdot 2$$

$$\left(\sqrt{3}\cos x-\sin x\right)^2+3\left(\sqrt{3}\cos x-\sin x\right)+2=0$$

$$t=\sqrt{3}\cos x-\sin x$$

$$t^2+3t+2=0$$

$$t_{1,2}=\frac{-3\pm\sqrt{9-8}}{2}=\frac{-3\pm 1}{2}$$

$$t_1=-2 \implies \sqrt{3}\cos x-\sin x=-2$$

$$2\sin\left(\frac{\pi}{3}-x\right)=-2$$

$$\sin\left(\frac{\pi}{3}-x\right)=-1$$

$$\frac{\pi}{3}-x=\frac{3\pi}{2}+2k\pi$$

$$-x=\frac{7\pi}{6}+2k\pi$$

$$x=-\frac{7\pi}{6}+2k\pi, \quad k\in\mathbf{Z}$$

$$x=\frac{5\pi}{6}+2k\pi, \quad k\in\mathbf{Z}$$

$$t_2=-1 \implies \sqrt{3}\cos x-\sin x=-1$$

$$2\sin\left(\frac{\pi}{3}-x\right)=-1$$

$$\sin\left(\frac{\pi}{3}-x\right)=-\frac{1}{2}$$

$$\frac{\pi}{3}-x=\frac{7\pi}{6}+2k\pi \qquad \frac{\pi}{3}-x=\frac{11\pi}{6}+2k\pi$$

$$-x=\frac{5\pi}{6}+2k\pi \qquad -x=\frac{9\pi}{6}+2k\pi$$

$$x=-\frac{5\pi}{6}+2k\pi \qquad x=-\frac{3\pi}{2}+2k\pi$$

$$x=-\frac{5\pi}{6}+2k\pi \qquad x=\frac{\pi}{2}+2k\pi$$

unija rješenja daje $x_1=\pm\frac{5\pi}{6}+2k\pi, \quad x_2=\frac{\pi}{2}+2k\pi, \quad k\in\mathbf{Z};$

4)

$$2\sin^2\left(\frac{\pi}{3}+x\right)-3\cos\left(\frac{\pi}{6}-x\right)+1=0$$

$$2\left(\sin\frac{\pi}{3}\cos x+\cos\frac{\pi}{3}\sin x\right)^2-3\left(\cos\frac{\pi}{6}\cos x+\sin\frac{\pi}{6}\sin x\right)+1=0$$

$$2\left(\frac{\sqrt{3}}{2}\cos x+\frac{1}{2}\sin x\right)^2-3\left(\frac{\sqrt{3}}{2}\cos x+\frac{1}{2}\sin x\right)+1=0$$

$$t=\frac{\sqrt{3}}{2}\cos x+\frac{1}{2}\sin x$$

$$2t^2-3t+1=0$$

$$t_{1,2}=\frac{3\pm\sqrt{9-8}}{4}=\frac{3\pm 1}{4}$$

$$t_1=1 \implies \frac{\sqrt{3}}{2}\cos x+\frac{1}{2}\sin x=1$$

$$\sin\left(\frac{\pi}{3}+x\right)=1$$

$$\frac{\pi}{3}+x=\frac{\pi}{2}+2k\pi$$

$$x=\frac{\pi}{6}+2k\pi$$

$$t_2=\frac{1}{2} \implies \frac{\sqrt{3}}{2}\cos x+\frac{1}{2}\sin x=\frac{1}{2}$$

$$\sin\left(\frac{\pi}{3}+x\right)=\frac{1}{2}$$

$$\frac{\pi}{3}+x=\frac{\pi}{6}+2k\pi \qquad \frac{\pi}{3}+x=\frac{5\pi}{6}+2k\pi$$

$$x=-\frac{\pi}{6}+2k\pi \qquad x=\frac{\pi}{2}+2k\pi$$

unija rješenja daje $x_1=\pm\frac{\pi}{6}+2k\pi,\quad x_2=\frac{\pi}{2}+2k\pi,\quad k\in\mathbf{Z};$