Zadatak 11.

1) $\sin^2 x - 8\sin x \cdot \cos x + 7\cos^2 x = 0$;
2) $\cos^2 x - 3\sin x \cdot \cos x + 1 = 0$;
3) $\sin^2 x + 9\cos^2 x = 5\sin 2x$;
4) $\cos^2 x - 7\sin^2 x = 3\sin 2x$;
5) $\sin^4 x - 5\sin^2 x \cdot \cos^2 x + 4\cos^4 x = 0$;
6) $2\sin^2 x - 3\sin x \cdot \cos x + \cos^2 x = 3$;
7) $8\sin^2 x + \sin x \cdot \cos x + \cos^2 x = 4$;

Rješenje. **1)**

$$\sin^2 x - 8\sin x \cdot \cos x + 7\cos^2 x = 0 \quad / : \cos^2 x$$
$$\operatorname{tg}^2 x - 8\operatorname{tg} x + 7 = 0$$
$$(\operatorname{tg} x)_{1,2} = \frac{8 \pm \sqrt{64 - 28}}{2} = \frac{8 \pm 6}{2}$$
$$(\operatorname{tg} x)_1 = 7 \implies x_1 = \operatorname{arc\,tg} 7 + k\pi, \quad k \in \mathbf{Z}$$
$$(\operatorname{tg} x)_2 = 1 \implies x_2 = \frac{\pi}{4} + k\pi, \quad k \in \mathbf{Z}$$

2)

$$\cos^2 x - 3\sin x \cdot \cos x + 1 = 0$$
$$\cos^2 x - 3\sin x \cdot \cos x + \sin^2 x + \cos^2 x = 0$$
$$2\cos^2 x - 3\sin x \cdot \cos x + \sin^2 x = 0 \quad / : \cos^2 x$$
$$2 - 3\operatorname{tg} x + \operatorname{tg}^2 x = 0$$
$$(\operatorname{tg} x)_{1,2} = \frac{3 \pm \sqrt{9 - 8}}{2} = \frac{3 \pm 1}{2}$$
$$(\operatorname{tg} x)_1 = 2 \implies x_1 = \operatorname{arc\,tg} 2 + k\pi, \quad k \in \mathbf{Z}$$
$$(\operatorname{tg} x)_2 = 1 \implies x_2 = \frac{\pi}{4} + k\pi, \quad k \in \mathbf{Z}$$

3)

$$\sin^2 x + 9\cos^2 x = 5\sin 2x$$
$$\sin^2 x + 9\cos^2 x - 10\sin x\cos x = 0 \quad / : \cos^2 x$$
$$\operatorname{tg}^2 x + 9 - 10\operatorname{tg} x = 0$$
$$(\operatorname{tg} x)_{1,2} = \frac{10 \pm \sqrt{100 - 36}}{2} = \frac{10 \pm 8}{2}$$
$$(\operatorname{tg} x)_1 = 9 \implies x_1 = \operatorname{arc\,tg} 9 + k\pi, \quad k \in \mathbf{Z}$$
$$(\operatorname{tg} x)_2 = 1 \implies x_2 = \frac{\pi}{4} + k\pi, \quad k \in \mathbf{Z}$$

4)

$$\cos^2 x - 7\sin^2 x = 3\sin 2x$$
$$\cos^2 x - 7\sin^2 x - 6\sin x\cos x = 0 \quad / : \cos^2 x$$
$$1 - 7\,\mathrm{tg}^2 x - 6\,\mathrm{tg}\, x = 0$$
$$(\mathrm{tg}\, x)_{1,2} = \frac{6 \pm \sqrt{36+28}}{-14} = \frac{6 \pm 8}{-14}$$
$$(\mathrm{tg}\, x)_1 = 1 \implies x_1 = \frac{\pi}{4} + k\pi, \quad k \in \mathbf{Z}$$
$$(\mathrm{tg}\, x)_2 = \frac{1}{7} \implies x_2 = \mathrm{arc\,tg}\,\frac{1}{7} + k\pi, \quad k \in \mathbf{Z}$$

5)

$$\sin^4 x - 5\sin^2 x \cdot \cos^2 x + 4\cos^4 x = 0 \quad / : \cos^2 x$$
$$\mathrm{tg}^4 x - 5\,\mathrm{tg}^2 x + 4 = 0$$
$$(\mathrm{tg}^2 x)_{1,2} = \frac{5 \pm \sqrt{25-16}}{2} = \frac{5 \pm 3}{2}$$
$$(\mathrm{tg}^2 x)_1 = 4 \implies (\mathrm{tg}\, x)_1 = \pm 2 \implies x_1 = \mathrm{arc\,tg}(\pm 2) + k\pi, \quad k \in \mathbf{Z}$$
$$(\mathrm{tg}^2 x)_2 = 1 \implies (\mathrm{tg}\, x)_2 = \pm 1 \implies x_2 = \pm\frac{\pi}{4} + k\pi, \quad k \in \mathbf{Z}$$

6)

$$2\sin^2 x - 3\sin x \cdot \cos x + \cos^2 x = 3$$
$$2\sin^2 x - 3\sin x \cdot \cos x + \cos^2 x = 3(\sin^2 x + \cos^2 x)$$
$$-\sin^2 x - 3\sin x \cdot \cos x - 2\cos^2 x = 0 \quad / : \cos^2 x$$
$$-\mathrm{tg}^2 x - 3\,\mathrm{tg}\, x - 2 = 0 \quad / \cdot (-1)$$
$$\mathrm{tg}^2 x + 3\,\mathrm{tg}\, x + 2 = 0$$
$$(\mathrm{tg}\, x)_{1,2} = \frac{-3 \pm \sqrt{9-8}}{2} = \frac{-3 \pm 1}{2}$$
$$(\mathrm{tg}\, x)_1 = -2 \implies x_1 = \mathrm{arc\,tg}(-2) + k\pi, \quad k \in \mathbf{Z}$$
$$(\mathrm{tg}\, x)_2 = -1 \implies x_2 = -\frac{\pi}{4} + k\pi, \quad k \in \mathbf{Z};$$

7)

$$8\sin^2 x + \sin x \cdot \cos x + \cos^2 x = 4$$
$$8\sin^2 x + \sin x \cdot \cos x + \cos^2 x = 4(\sin^2 x + \cos^2 x)$$
$$4\sin^2 x + \sin x \cdot \cos x - 3\cos^2 x = 0 \quad / : \cos^2 x$$
$$4\,\mathrm{tg}^2 x + \mathrm{tg}\, x - 3 = 0$$
$$(\mathrm{tg}\, x)_{1,2} = \frac{-1 \pm \sqrt{1+48}}{8} = \frac{-1 \pm 7}{8}$$
$$(\mathrm{tg}\, x)_1 = -1 \implies x_1 = -\frac{\pi}{4} + k\pi, \quad k \in \mathbf{Z};$$
$$(\mathrm{tg}\, x)_2 = \frac{3}{4} \implies x_2 = \mathrm{arc\,tg}\,\frac{3}{4} + k\pi, \quad k \in \mathbf{Z}$$