Zadatak 13. Rastavljanjem na faktore riješi jednadžbe:

1) $1+\sin x\cdot\cos 2x=\sin x+\cos 2x$;

2) $\operatorname{ctg} x+\cos x=1+\operatorname{ctg} x\cdot\cos x$;

3) $\operatorname{tg}^3 x+\operatorname{tg}^2 x-2\operatorname{tg} x-2=0$;

4) $4\sin x\cdot\cos x-1=2(\sin x-\cos x)$;

5) $12\sin^2 x\cdot\operatorname{tg}^2 x+3=4\sin^2 x+9\operatorname{tg}^2 x$.

Rješenje. **1)**

$$1+\sin x\cdot\cos 2x=\sin x+\cos 2x$$
$$1-\sin x-\cos 2x+\sin x\cdot\cos 2x=0$$
$$1-\sin x-\cos 2x(1-\sin x)=0$$
$$(1-\sin x)(1-\cos 2x)=0$$
$$1^\circ\quad 1-\sin x=0$$
$$\sin x=1\implies x_1=\frac{\pi}{2}+2k\pi,\quad k\in\mathbf{Z}$$
$$2^\circ\quad 1-\cos 2x=0$$
$$\cos 2x=1\implies 2x_2=2k\pi,\quad x_2=k\pi,\quad k\in\mathbf{Z}$$

2)

$$\operatorname{ctg} x+\cos x=1+\operatorname{ctg} x\cdot\cos x$$
$$\operatorname{ctg} x+\cos x-1-\operatorname{ctg} x\cdot\cos x=0$$
$$\cos x-1-\operatorname{ctg} x(\cos x-1)=0$$
$$(\cos x-1)(1-\operatorname{ctg} x)=0$$
$$1^\circ\quad \cos x-1=0$$
$$\cos x=1\implies x_1=2k\pi,\quad k\in\mathbf{Z}$$
$$2^\circ\quad 1-\operatorname{ctg} x=0$$
$$\operatorname{ctg} x=1\implies x_2=\frac{\pi}{4}+k\pi,\quad k\in\mathbf{Z}$$

3)

$$\operatorname{tg}^3 x+\operatorname{tg}^2 x-2\operatorname{tg} x-2=0$$
$$\operatorname{tg}^2 x(\operatorname{tg} x+1)-2(\operatorname{tg} x+1)=0$$
$$(\operatorname{tg} x+1)(\operatorname{tg}^2 x-2)=0$$
$$1^\circ\quad \operatorname{tg} x+1=0$$
$$\operatorname{tg} x=-1\implies x_1=\frac{3\pi}{4}+k\pi,\quad k\in\mathbf{Z}$$
$$2^\circ\quad \operatorname{tg}^2 x-2=0$$
$$\operatorname{tg}^2 x=2,\quad \operatorname{tg} x=\pm\sqrt{2}\implies x_2=\operatorname{arc\,tg}(\pm\sqrt{2})+k\pi,\quad k\in\mathbf{Z}$$

4)

$$4\sin x\cdot\cos x-1=2(\sin x-\cos x)$$
$$4\sin x\cdot\cos x-1-2\sin x+2\cos x=0$$
$$2\cos x(2\sin x+1)-(1+2\sin x)=0$$
$$(2\sin x+1)(2\cos x-1)=0$$

1° $2\sin x+1=0$

$$2\sin x=-1,\quad \sin x=-\frac{1}{2}\implies x_1=\frac{5\pi}{6}+2k\pi,\quad x_2=\frac{7\pi}{6}+2k\pi,\quad k\in\mathbf{Z}$$

2° $2\cos x-1=0$

$$2\cos x=1,\quad \cos x=\frac{1}{2}\implies x_3=\pm\frac{\pi}{3}+2k\pi,\quad k\in\mathbf{Z}$$

5)

$$12\sin^2 x\cdot\operatorname{tg}^2 x+3=4\sin^2 x+9\operatorname{tg}^2 x$$
$$4\sin^2 x(3\operatorname{tg}^2 x-1)-3(3\operatorname{tg}^2 x-1)=0$$
$$(3\operatorname{tg}^2 x-1)(4\sin^2 x-3)=0$$

1° $3\operatorname{tg}^2 x-1=0$

$$3\operatorname{tg}^2 x=1,\quad \operatorname{tg}^2 x=\frac{1}{3},\quad \operatorname{tg} x=\pm\frac{\sqrt{3}}{3}\implies x_1=\pm\frac{\pi}{6}+k\pi,\quad k\in\mathbf{Z}$$

2° $4\sin^2 x-3=0$

$$4\sin^2 x=3,\quad \sin^2 x=\frac{3}{4},\quad \sin x=\pm\frac{\sqrt{3}}{2}$$
$$\implies x_2\in\left\{\frac{\pi}{3}+2k\pi,\ \frac{2\pi}{3}+2k\pi,\ \frac{4\pi}{3}+2k\pi,\ \frac{5\pi}{3}+2k\pi\right\},$$
$$\implies\quad x_2=\pm\frac{\pi}{3}+k\pi,\quad k\in\mathbf{Z}$$