Zadatak 14. Primjenjujući adicijski teorem riješi sljedeće jednadžbe:

1) $\sin x \cdot \cos \frac{x}{2} + \cos x \cdot \sin \frac{x}{2} = 0$;

2) $\sin x \cdot \cos 2x + \cos x \cdot \sin 2x = \frac{\sqrt{3}}{2}$;

3) $\sin\left(\frac{\pi}{3} - x\right) \cdot \sin\left(\frac{\pi}{6} - x\right) + \sin\left(\frac{\pi}{3} + x\right) \cdot \sin\left(\frac{\pi}{6} + x\right) = \frac{\sqrt{3}}{2}$;

4) $\sin\left(\frac{\pi}{6} + x\right) \cdot \cos\left(\frac{\pi}{3} - x\right) - \cos\left(\frac{\pi}{6} + x\right) \cdot \sin\left(\frac{\pi}{3} - x\right) = \frac{1}{2}$;

5) $\frac{\operatorname{tg} x + \operatorname{tg} 2x}{1 - \operatorname{tg} x \cdot \operatorname{tg} 2x} = \frac{1}{\sqrt{3}}$;

6) $3 + \sqrt{3} \operatorname{tg} x = \sqrt{3} - 3 \operatorname{tg} x$.

Rješenje. **1)**

$$\sin x \cdot \cos \frac{x}{2} + \cos x \cdot \sin \frac{x}{2} = 0$$

$$\sin\left(x + \frac{x}{2}\right) = 0$$

$$\sin \frac{3x}{2} = 0 \implies \frac{3x}{2} = k\pi, \quad x = \frac{2k\pi}{3}, \quad k \in \mathbf{Z}$$

2)

$$\sin x \cdot \cos 2x + \cos x \cdot \sin 2x = \frac{\sqrt{3}}{2}$$

$$\sin(x + 2x) = \frac{\sqrt{3}}{2}$$

$$\sin 3x = \frac{\sqrt{3}}{2} \implies \begin{cases} 3x_1 = \frac{\pi}{3} + 2k\pi, & x_1 = \frac{\pi}{9} + \frac{2k\pi}{3}, \quad k \in \mathbf{Z} \\ 3x_2 = \frac{2\pi}{3} + 2k\pi, & x_2 = \frac{2\pi}{9} + \frac{2k\pi}{3}, \quad k \in \mathbf{Z} \end{cases}$$

3)

$$\sin\left(\frac{\pi}{3}-x\right)\cdot\sin\left(\frac{\pi}{6}-x\right)+\sin\left(\frac{\pi}{3}+x\right)\cdot\sin\left(\frac{\pi}{6}+x\right)=\frac{\sqrt{3}}{2}$$

$$\left(\sin\frac{\pi}{3}\cos x-\cos\frac{\pi}{3}\sin x\right)\cdot\left(\sin\frac{\pi}{6}\cos x-\cos\frac{\pi}{6}\sin x\right)$$

$$+\left(\sin\frac{\pi}{3}\cos x+\cos\frac{\pi}{3}\sin x\right)\cdot\left(\sin\frac{\pi}{6}\cos x+\cos\frac{\pi}{6}\sin x\right)=\frac{\sqrt{3}}{2}$$

$$\left(\frac{\sqrt{3}}{2}\cos x-\frac{1}{2}\sin x\right)\cdot\left(\frac{1}{2}\cos x-\frac{\sqrt{3}}{2}\sin x\right)+\left(\frac{\sqrt{3}}{2}\cos x+\frac{1}{2}\sin x\right)\cdot\left(\frac{1}{2}\cos x+\frac{\sqrt{3}}{2}\sin x\right)=\frac{\sqrt{3}}{2}$$

$$\frac{\sqrt{3}}{4}\cos^2 x-\frac{3}{4}\sin x\cos x-\frac{1}{4}\sin x\cos x+\frac{\sqrt{3}}{4}\sin^2 x+\frac{\sqrt{3}}{4}\cos^2 x+\frac{1}{4}\sin x\cos x+\frac{\sqrt{3}}{4}\sin^2 x=\frac{\sqrt{3}}{2}$$

$$\frac{\sqrt{3}}{2}\cos^2 x+\frac{\sqrt{3}}{2}\sin^2 x=\frac{\sqrt{3}}{2}\quad\Big/:\frac{\sqrt{3}}{2}$$

$\sin^2 x+\cos^2 x=1,\quad$ vrijedi $\forall x\in\mathbf{R}$

Drugi način:

$$\sin\left(\frac{\pi}{3}-x\right)\cdot\sin\left(\frac{\pi}{6}-x\right)+\sin\left(\frac{\pi}{3}+x\right)\cdot\sin\left(\frac{\pi}{6}+x\right)=\frac{\sqrt{3}}{2}$$

$$\cos\left(\frac{\pi}{2}-\frac{\pi}{3}+x\right)\cdot\sin\left(\frac{\pi}{6}-x\right)+\cos\left(\frac{\pi}{2}-\frac{\pi}{3}-x\right)\cdot\sin\left(\frac{\pi}{6}+x\right)=\frac{\sqrt{3}}{2}$$

$$\cos\left(\frac{\pi}{6}+x\right)\cdot\sin\left(\frac{\pi}{6}-x\right)+\cos\left(\frac{\pi}{6}-x\right)\cdot\sin\left(\frac{\pi}{6}+x\right)=\frac{\sqrt{3}}{2}$$

$$\sin\left(\frac{\pi}{6}-x+\frac{\pi}{6}+x\right)=\frac{\sqrt{3}}{2}$$

$$\sin\frac{\pi}{3}=\frac{\sqrt{3}}{2},\quad \text{vrijedi } \forall x\in\mathbf{R}$$

4)

$$\sin\left(\frac{\pi}{6}+x\right)\cdot\cos\left(\frac{\pi}{3}-x\right)-\cos\left(\frac{\pi}{6}+x\right)\cdot\sin\left(\frac{\pi}{3}-x\right)=\frac{1}{2}$$

$$\sin\left(\frac{\pi}{6}+x-\frac{\pi}{3}+x\right)=\frac{1}{2}$$

$$\sin\left(2x-\frac{\pi}{6}\right)=\frac{1}{2}$$

1° $\quad 2x-\frac{\pi}{6}=\frac{\pi}{3}+2k\pi$

$2x=\frac{\pi}{2}+2k\pi$

$x_1=\frac{\pi}{4}+k\pi,\quad k\in\mathbf{Z}$

2° $\quad 2x-\frac{\pi}{6}=\frac{2\pi}{3}+2k\pi$

$2x=\frac{5\pi}{6}+2k\pi$

$x_2=\frac{5\pi}{12}+k\pi,\quad k\in\mathbf{Z}$

5)

$$\frac{\operatorname{tg} x + \operatorname{tg} 2x}{1 - \operatorname{tg} x \cdot \operatorname{tg} 2x} = \frac{1}{\sqrt{3}}$$

$$\operatorname{tg}(x + 2x) = \frac{\sqrt{3}}{3}$$

$$\operatorname{tg} 3x = \frac{\sqrt{3}}{3} \implies 3x = \frac{\pi}{6} + k\pi, \quad x = \frac{\pi}{18} + \frac{k\pi}{3}, \quad k \in \mathbf{Z}$$

6)

$$3 + \sqrt{3} \operatorname{tg} x = \sqrt{3} - 3 \operatorname{tg} x \quad / : \sqrt{3}$$

$$\sqrt{3} + \operatorname{tg} x = 1 - \sqrt{3} \operatorname{tg} x \quad / : (1 - \sqrt{3} \operatorname{tg} x)$$

$$\frac{\sqrt{3} + \operatorname{tg} x}{1 - \sqrt{3} \operatorname{tg} x} = 1$$

$$\frac{\operatorname{tg} \frac{\pi}{3} + \operatorname{tg} x}{1 - \operatorname{tg} \frac{\pi}{3} \operatorname{tg} x} = 1$$

$$\operatorname{tg}\left(\frac{\pi}{3} + x\right) = 1 \implies x + \frac{\pi}{3} = \frac{\pi}{4} + k\pi, \quad x = -\frac{\pi}{12} + k\pi, \quad k \in \mathbf{Z}$$