Zadatak 16. Primjenjujući identitete (univerzalna zamjena)

$$\sin x = \frac{2\operatorname{tg}\frac{x}{2}}{1+\operatorname{tg}^2\frac{x}{2}}, \quad \cos x = \frac{1-\operatorname{tg}^2\frac{x}{2}}{1+\operatorname{tg}^2\frac{x}{2}}, \quad x \neq k\pi,$$

riješi sljedeće jednadžbe:

1) $\sin x - 3\cos x = 1$;
2) $\sin x + 7\cos x + 7 = 0$;
3) $2\sin 2x - 5\cos 2x = -3$;
4) $3\sin 2x + 2\cos 2x = 3$;
5) $2\sin x + 9\cos x = 7$.

Rješenje. **1)**

$$\sin x - 3\cos x = 1$$

$$\frac{2\operatorname{tg}\frac{x}{2}}{1+\operatorname{tg}^2\frac{x}{2}} - 3 = \frac{1-\operatorname{tg}^2\frac{x}{2}}{1+\operatorname{tg}^2\frac{x}{2}} = 1 \quad \Big/ \cdot \left(1+\operatorname{tg}^2\frac{x}{2}\right)$$

$$2\operatorname{tg}\frac{x}{2} - 3 + 3\operatorname{tg}^2\frac{x}{2} = 1 + \operatorname{tg}^2\frac{x}{2}$$

$$2\operatorname{tg}^2\frac{x}{2} + 2\operatorname{tg}\frac{x}{2} - 2 = 0$$

$$\left(\operatorname{tg}\frac{x}{2}\right)_{1,2} = \frac{-1 \pm \sqrt{1+8}}{2} = \frac{-1\pm 3}{2}$$

$$\left(\operatorname{tg}\frac{x}{2}\right)_1 = -2 \implies \frac{x_1}{2} = \operatorname{arc\,tg}(-2) + k\pi, \quad x_1 = 2\operatorname{arc\,tg}(-2) + 2k\pi, \quad k \in \mathbf{Z}$$

$$\left(\operatorname{tg}\frac{x}{2}\right)_2 = 1 \implies \frac{x_2}{2} = \frac{\pi}{4} + k\pi, \quad x_2 = \frac{\pi}{2} + 2k\pi, \quad k \in \mathbf{Z}$$

2)

$$\sin x + 7\cos x + 7 = 0$$

$$\frac{2\operatorname{tg}\frac{x}{2}}{1+\operatorname{tg}^2\frac{x}{2}} + 7\cdot\frac{1-\operatorname{tg}^2\frac{x}{2}}{1+\operatorname{tg}^2\frac{x}{2}} + 7 = 0 \quad \Big/ \cdot \left(1+\operatorname{tg}^2\frac{x}{2}\right)$$

$$2\operatorname{tg}\frac{x}{2} + 7 - 7\operatorname{tg}^2\frac{x}{2} + 7 + 7\operatorname{tg}^2\frac{x}{2} = 0$$

$$2\operatorname{tg}\frac{x}{2} + 14 = 0$$

$$\operatorname{tg}\frac{x}{2} = -7 \implies \frac{x}{2} = -\operatorname{arc\,tg}7 + k\pi, \quad x = -2\operatorname{arc\,tg}7 + 2k\pi, \quad k \in \mathbf{Z}$$

3)

$$2\sin 2x - 5\cos 2x = -3$$

$$2\cdot\frac{2\operatorname{tg}x}{1+\operatorname{tg}^2 x} - 5\cdot\frac{1-\operatorname{tg}^2 x}{1+\operatorname{tg}^2 x} = -3 \quad /\cdot(1+\operatorname{tg}^2 x)$$

$$4\operatorname{tg}x - 5 + 5\operatorname{tg}^2 x = -3 - 3\operatorname{tg}^2 x$$

$$8\operatorname{tg}^2 x + 4\operatorname{tg}x - 2 = 0 \quad /:2$$

$$4\operatorname{tg}^2 x + 2\operatorname{tg}x - 1 = 0$$

$$(\operatorname{tg}x)_{1,2} = \frac{-2\pm\sqrt{4+16}}{8} = \frac{-2\pm\sqrt{20}}{8} = \frac{2(-1\pm\sqrt{5})}{8} = \frac{-1\pm\sqrt{5}}{4}$$

$$(\operatorname{tg}x)_1 = -0.8090 \implies x_1 = -0.6802 + k\pi, \quad k\in\mathbf{Z}$$

$$(\operatorname{tg}x)_2 = 0.3090 \implies x_2 = 0.2997 + k\pi, \quad k\in\mathbf{Z}$$

4)

$$3\sin 2x + 2\cos 2x = 3$$

$$3\cdot\frac{2\operatorname{tg}x}{1+\operatorname{tg}^2 x} + 2\cdot\frac{1-\operatorname{tg}^2 x}{1+\operatorname{tg}^2 x} = 3 \quad /\cdot(1+\operatorname{tg}^2 x)$$

$$6\operatorname{tg}x + 2 - 2\operatorname{tg}^2 x = 3 + 3\operatorname{tg}^2 x$$

$$-5\operatorname{tg}^2 x + 6\operatorname{tg}x - 1 = 0 \quad /\cdot(-1)$$

$$5\operatorname{tg}^2 x - 6\operatorname{tg}x + 1 = 0$$

$$(\operatorname{tg}x)_{1,2} = \frac{6\pm\sqrt{36-20}}{10} = \frac{6\pm 4}{10} = \frac{3\pm 2}{5}$$

$$(\operatorname{tg}x)_1 = 1 \implies x_1 = \frac{\pi}{4} + k\pi, \quad k\in\mathbf{Z}$$

$$(\operatorname{tg}x)_2 = \frac{1}{5} \implies x_2 = \operatorname{arc\,tg}\frac{1}{5} + k\pi = 0.1974 + k\pi, \quad k\in\mathbf{Z}$$

5)

$$2\sin x + 9\cos x = 7$$

$$2\cdot\frac{2\operatorname{tg}\frac{x}{2}}{1+\operatorname{tg}^2\frac{x}{2}} + 9\cdot\frac{1-\operatorname{tg}^2\frac{x}{2}}{1+\operatorname{tg}^2\frac{x}{2}} = 7 \quad \Big/\cdot\left(1+\operatorname{tg}^2\frac{x}{2}\right)$$

$$4\operatorname{tg}\frac{x}{2} + 9 - 9\operatorname{tg}^2\frac{x}{2} = 7 + 7\operatorname{tg}^2\frac{x}{2}$$

$$-16\operatorname{tg}^2\frac{x}{2} + 4\operatorname{tg}\frac{x}{2} + 2 = 0$$

$$\left(\operatorname{tg}\frac{x}{2}\right)_{1,2} = \frac{2\pm\sqrt{4+32}}{16} = \frac{2\pm 6}{16}$$

$$\left(\operatorname{tg}\frac{x}{2}\right)_1 = \frac{1}{2} \implies \frac{x_1}{2} = \operatorname{arc\,tg}\frac{1}{2} + k\pi, \quad x_1 = 2\operatorname{arc\,tg}\frac{1}{2} + 2k\pi, \quad k\in\mathbf{Z}$$

$$\left(\operatorname{tg}\frac{x}{2}\right)_2 = -\frac{1}{4} \implies \frac{x_2}{2} = -\operatorname{arc\,tg}\frac{1}{4} + k\pi, \quad x_2 = -2\operatorname{arc\,tg}\frac{1}{4} + 2k\pi, \quad k\in\mathbf{Z}$$