Zadatak 17. Riješi jednadžbe:

1) $\sin x + \cos x = 1$;

2) $\sin x + \sqrt{3}\cos x = 2$;

3) $\sqrt{3}\sin x - \cos x = 2$;

4) $\cos 2x - \sqrt{3}\sin 2x = \sqrt{3}$;

5) $\sqrt{3}\sin x - \cos x = \sqrt{2}$;

6) $\sqrt{3}\sin 3x + \cos 3x = 1$.

Rješenje. **1)**

$$\sin x + \cos x = 1 \quad \Big/ \cdot \frac{\sqrt{2}}{2}$$
$$\frac{\sqrt{2}}{2}\sin x + \frac{\sqrt{2}}{2}\cos x = \frac{\sqrt{2}}{2}$$
$$\cos\frac{\pi}{4}\sin x + \sin\frac{\pi}{4}\cos x = \frac{\sqrt{2}}{2}$$
$$\sin\left(\frac{\pi}{4} + x\right) = \frac{\sqrt{2}}{2}$$
$$1^\circ \quad x_1 + \frac{\pi}{4} = \frac{\pi}{4} + 2k\pi, \quad x_1 = 2k\pi, \quad k \in \mathbf{Z}$$
$$2^\circ \quad x_2 + \frac{\pi}{4} = \frac{3\pi}{4} + 2k\pi, \quad x_2 = \frac{\pi}{2} + 2k\pi, \quad k \in \mathbf{Z}$$

2)

$$\sin x + \sqrt{3}\cos x = 2 \quad / : 2$$
$$\frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x = 1$$
$$\cos\frac{\pi}{3}\sin x + \sin\frac{\pi}{3}\cos x = 1$$
$$\sin\left(x + \frac{\pi}{3}\right) = 1 \implies x + \frac{\pi}{3} = \frac{\pi}{2} + 2k\pi, \quad x = \frac{\pi}{6} + 2k\pi, \quad k \in \mathbf{Z}$$

3)

$$\sqrt{3}\sin x - \cos x = 2 \quad / : 2$$
$$\frac{\sqrt{3}}{2}\sin x - \frac{1}{2}\cos x = 1$$
$$\cos\frac{\pi}{6}\sin x - \sin\frac{\pi}{6}\cos x = 1$$
$$\sin\left(x - \frac{\pi}{6}\right) = 1 \implies x - \frac{\pi}{6} = \frac{\pi}{2} + 2k\pi, \quad x = \frac{2\pi}{3} + 2k\pi, \quad k \in \mathbf{Z}$$

4)

$$\cos 2x - \sqrt{3}\sin 2x = \sqrt{3} \quad / : 2$$

$$\frac{1}{2}\cos 2x - \frac{\sqrt{3}}{2}\sin 2x = \frac{\sqrt{3}}{2}$$

$$\cos\frac{\pi}{3}\cos 2x - \sin\frac{\pi}{3}\sin 2x = \frac{\sqrt{3}}{2}$$

$$\cos\left(\frac{\pi}{3} + 2x\right) = \frac{\sqrt{3}}{2}$$

$$1^\circ \quad 2x_1 + \frac{\pi}{3} = \frac{\pi}{6} + 2k\pi, \quad 2x_1 = -\frac{\pi}{6} + 2k\pi, \quad x_1 = -\frac{\pi}{12} + k\pi, \quad k \in \mathbf{Z}$$

$$2^\circ \quad 2x_2 + \frac{\pi}{3} = -\frac{\pi}{6} + 2k\pi, \quad 2x_2 = -\frac{\pi}{2} + 2k\pi, \quad x_2 = -\frac{\pi}{4} + k\pi, \quad k \in \mathbf{Z}$$

5)

$$\sqrt{3}\sin x - \cos x = \sqrt{2} \quad / : 2$$

$$\frac{\sqrt{3}}{2}\sin x - \frac{1}{2}\cos x = \frac{\sqrt{2}}{2}$$

$$\cos\frac{\pi}{6}\sin x - \sin\frac{\pi}{6}\cos x = \frac{\sqrt{2}}{2}$$

$$\sin\left(x - \frac{\pi}{6}\right) = \frac{\sqrt{2}}{2}$$

$$1^\circ \quad x - \frac{\pi}{6} = \frac{\pi}{4} + 2k\pi, \quad x_1 = \frac{5\pi}{12} + 2k\pi, \quad k \in \mathbf{Z}$$

$$2^\circ \quad x - \frac{\pi}{6} = \frac{3\pi}{4} + 2k\pi, \quad x_2 = \frac{11\pi}{12} + 2k\pi, \quad k \in \mathbf{Z}$$

6)

$$\sqrt{3}\sin 3x + \cos 3x = 1 \quad / : 2$$

$$\frac{\sqrt{3}}{2}\sin 3x + \frac{1}{2}\cos 3x = \frac{1}{2}$$

$$\sin\frac{\pi}{3}\sin 3x + \cos\frac{\pi}{3}\cos 3x = \frac{1}{2}$$

$$\cos\left(3x - \frac{\pi}{3}\right) = \frac{1}{2}$$

$$1^\circ \quad 3x_1 - \frac{\pi}{3} = \frac{\pi}{3} + 2k\pi, \quad 3x_1 = \frac{2\pi}{3} + 2k\pi, \quad x_1 = \frac{2\pi}{9} + \frac{2k\pi}{3}, \quad k \in \mathbf{Z}$$

$$2^\circ \quad 3x_2 - \frac{\pi}{3} = -\frac{\pi}{3} + 2k\pi, \quad 3x_2 = 2k\pi, \quad x_2 = \frac{2k\pi}{3}, \quad k \in \mathbf{Z}$$