Zadatak 18. Riješi jednadžbe:

1) $(\cos x - \sin x)^2 = \sin 2x$;

2) $(\sin x + \cos x)^2 = 1 + \cos x$;

3) $(\sin x + \cos x)^2 = 1 - \cos 2x$;

4) $\sin x + \cos x = \sqrt{2} \cos 2x$;

5) $\sin x + \sin 2x = \cos x + 2\cos^2 x$.

Rješenje. **1)**

$(\cos x - \sin x)^2 = \sin 2x$

$\cos^2 x - 2\sin x \cos x + \sin^2 x = \sin 2x$

$1 - \sin 2x - \sin 2x = 0$

$2\sin 2x = 1$

$\sin 2x = \frac{1}{2}$

$1^\circ \quad 2x_1 = \frac{\pi}{6} + 2k\pi, \quad x_1 = \frac{\pi}{12} + k\pi, \quad k \in \mathbf{Z}$

$2^\circ \quad 2x_2 = \frac{5\pi}{6} + 2k\pi, \quad x_2 = \frac{5\pi}{12} + k\pi, \quad k \in \mathbf{Z}$

2)

$(\sin x + \cos x)^2 = 1 + \cos x$

$\sin^2 x + 2\sin x \cos x + \cos^2 x = 1 + \cos x$

$1 + 2\sin x \cos x - 1 - \cos x = 0$

$\cos x(2\sin x - 1) = 0$

$1^\circ \quad \cos x = 0 \implies x_1 = \frac{2k+1}{2}\pi, \quad k \in \mathbf{Z}$

$2^\circ \quad 2\sin x - 1 = 0, \quad \cos x = \frac{1}{2} \implies x_2 = \frac{\pi}{6} + 2k\pi, \quad x_3 = \frac{5\pi}{6} + 2k\pi, \quad k \in \mathbf{Z}$

3)

$(\sin x + \cos x)^2 = 1 - \cos 2x$

$\sin^2 x + 2\sin x \cos x + \cos^2 x = 1 - \cos 2x$

$1 + \sin 2x - 1 + \cos 2x = 0 \quad / : \cos 2x$

$\operatorname{tg} 2x + 1 = 0$

$\operatorname{tg} 2x = -1 \implies 2x = \frac{3\pi}{4} + k\pi, \quad x = \frac{3\pi}{8} + \frac{k\pi}{2}, \quad k \in \mathbf{Z}$

4)

$$\sin x + \cos x = \sqrt{2}\cos 2x$$
$$\sin x + \cos x = \sqrt{2}(\cos^2 x - \sin^2 x)$$
$$\sin x + \cos x - \sqrt{2}(\cos x - \sin x)(\cos x + \sin x)$$
$$(\cos x + \sin x)[1 - \sqrt{2}(\cos x - \sin x)]$$

1° $\quad \cos x + \sin x = 0 \quad / : \cos x$

$$\operatorname{tg} x + 1 = 0$$
$$\operatorname{tg} x = -1 \implies x_1 = -\frac{\pi}{4} + k\pi, \quad k \in \mathbf{Z}$$

2° $\quad 1 - \sqrt{2}(\cos x - \sin x) = 0$

$$\sqrt{2}(\cos x - \sin x) = 1$$
$$\cos x - \sin x = \frac{\sqrt{2}}{2} \quad / \cdot \frac{\sqrt{2}}{2}$$
$$\frac{\sqrt{2}}{2}\cos x - \frac{\sqrt{2}}{2}\sin x = \frac{1}{2}$$
$$\cos\frac{\pi}{4}\cos x - \sin\frac{\pi}{4}\sin x = \frac{1}{2}$$
$$\cos\left(\frac{\pi}{4} + x\right) = \frac{1}{2} \implies \frac{\pi}{4} + x_2 = \frac{\pi}{3} + 2k\pi, \quad x_2 = \frac{\pi}{12} + 2k\pi,$$
$$\frac{\pi}{4} + x_3 = -\frac{\pi}{3} + 2k\pi, \quad x_3 = -\frac{7\pi}{12} + 2k\pi = \frac{17\pi}{12} + 2k\pi,$$

5)

$$\sin x + \sin 2x = \cos x + 2\cos^2 x$$
$$\sin x - \cos x + 2\sin x\cos x - 2\cos^2 x = 0$$
$$\sin x - \cos x + 2\cos x(\sin x - \cos x) = 0$$
$$(\sin x - \cos x)(1 + 2\cos x) = 0$$

1° $\quad \sin x - \cos x = 0 \quad / : \cos x$

$$\operatorname{tg} x - 1 = 0$$
$$\operatorname{tg} x = 1 \implies x_1 = \frac{\pi}{4} + k\pi, \quad k \in \mathbf{Z}$$

2° $\quad 2\cos x + 1 = 0$

$$\cos x = -\frac{1}{2} \implies x_2 = \pm\frac{2\pi}{3} + 2k\pi, \quad k \in \mathbf{Z}$$