Zadatak 20.

Riješi jednadžbe:

1) $\sin 4x \cdot \cos 2x = \sin 5x \cdot \cos x$;
2) $\cos 5x \cdot \cos x = \cos 6x \cdot \cos 2x$;
3) $\sin 2x \cdot \sin 6x = \sin 3x \cdot \sin 5x$;
4) $\sin 3x \cdot \sin 7x = \sin x \cdot \sin 9x$;

Rješenje.

1)

$$\sin 4x \cdot \cos 2x = \sin 5x \cdot \cos x$$
$$\frac{1}{2}[\sin(4x+2x)+\sin(4x-2x)] = \frac{1}{2}[\sin(5x+x)+\sin(5x-x)]$$
$$\sin 6x + \sin 2x = \sin 6x + \sin 4x$$
$$\sin 2x - 2\sin 2x\cos 2x = 0$$
$$\sin 2x(1-2\cos 2x) = 0$$

$1^\circ \quad \sin 2x = 0, \quad 2x = k\pi, \quad x_1 = \frac{k\pi}{2}, \quad k \in \mathbf{Z}$

$2^\circ \quad 1 - 2\cos 2x = 0, \quad \cos 2x = \frac{1}{2}, \quad 2x = \pm\frac{\pi}{3} + 2k\pi, \quad x_{2,3} = \pm\frac{\pi}{6} + k\pi, \quad k \in \mathbf{Z}$

2)

$$\cos 5x \cdot \cos x = \cos 6x \cdot \cos 2x$$
$$\frac{1}{2}[\cos(5x+x)+\cos(5x-x)] = \frac{1}{2}[\cos(6x+2x)+\cos(6x-2x)]$$
$$\cos 6x + \cos 4x = \cos 8x + \cos 4x$$
$$\cos 6x - \cos 8x = 0$$
$$-2\sin\frac{6x+8x}{2}\sin\frac{6x-8x}{2} = 0$$
$$2\sin 7x \sin 2x = 0 \quad / : 2$$
$$\sin 7x \sin 2x = 0$$

$1^\circ \quad \sin 7x = 0, \quad 7x = k\pi, \quad x_1 = \frac{k\pi}{7}, \quad k \in \mathbf{Z}$

$2^\circ \quad \sin 2x = 0, \quad 2x = k\pi, \quad x_2 = \frac{k\pi}{2}, \quad k \in \mathbf{Z}$

x_2 je sadržan u x_1 pa je rješenje $x = \frac{k\pi}{7}$, $k \in \mathbf{Z}$;

3)

$\sin 2x \cdot \sin 6x = \sin 3x \cdot \sin 5x$

$\frac{1}{2}[\cos(2x-6x) - \cos(2x+6x)] = \frac{1}{2}[\cos(3x-5x) - \cos(3x+5x)]$

$\cos 4x - \cos 8x = \cos 2x - \cos 8x$

$\cos 4x - \cos 2x = 0$

$-2\sin\frac{4x+2x}{2}\sin\frac{4x-2x}{2} = 0 \quad / : (-2)$

$\sin 3x \sin x = 0$

$1^\circ \quad \sin 3x = 0, \quad 3x = k\pi, \quad x_1 = \frac{k\pi}{3}, \quad k \in \mathbf{Z}$

$2^\circ \quad \sin x = 0, \quad x_2 = k\pi, \quad k \in \mathbf{Z}$

x_2 je sadržan u x_1 pa je rješenje $x = \frac{k\pi}{3}$, $k \in \mathbf{Z}$;

4)

$\sin 3x \cdot \sin 7x = \sin x \cdot \sin 9x$

$\frac{1}{2}[\cos(3x-7x) - \cos(3x+7x)] = \frac{1}{2}[\cos(x-9x) - \cos(x+9x)]$

$\cos 4x - \cos 10x = \cos 8x - \cos 10x$

$\cos 4x - \cos 8x = 0$

$-2\sin\frac{4x+8x}{2}\sin\frac{4x-8x}{2} = 0$

$2\sin 6x \sin 2x = 0 \quad / : 2$

$\sin 6x \sin 2x = 0$

$1^\circ \quad \sin 6x = 0, \quad 6x = k\pi, \quad x_1 = \frac{k\pi}{6}, \quad k \in \mathbf{Z}$

$2^\circ \quad \sin 2x = 0, \quad 2x = k\pi, \quad x_2 = \frac{k\pi}{2}, \quad k \in \mathbf{Z}$

x_2 je sadržan u x_1 pa je rješenje $x = \frac{k\pi}{6}$, $k \in \mathbf{Z}$;