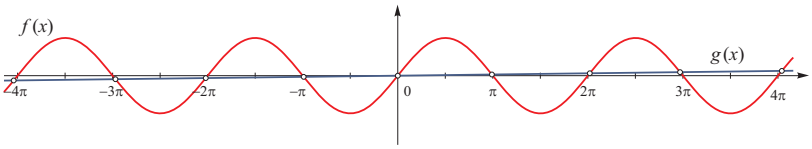
Zadatak 21. Koliko rješenja ima jednadžba:

1) $\sin x = \dfrac{x}{100}$; **2)** $\cos x = \dfrac{1}{x}$;

3) $\cos(\pi x) = \dfrac{x^2}{100}$; **4)** $\cos x = \dfrac{|x|}{10}$;

5) $x^3 = \sin(3x)$; **6)** $\cos(\pi x) = \log \dfrac{x}{100}$?

Rješenje. **1)** $\underbrace{\sin x}_{f(x)} = \underbrace{\dfrac{x}{100}}_{g(x)}$;



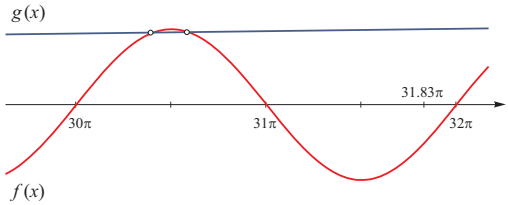
Za $x > 100$ je $\dfrac{|x|}{100} > 1$ pa se funkcije f i g za $x > 100$ ne sijeku.

Gledamo presjeke na intervalu $[-100, 100] \approx [-31.83\pi, 31.83\pi]$.

Na intevalu $[-30\pi, 30\pi]$ $\sin x$ ima 30 punih perioda. Na svakom od njih, izuzev $[-2\pi, 0]$, $[0, 2\pi]$ koji imaju zajedničku točku presjeka 0, ima po dvije točke presjeka.

Stoga ukupni broj točaka presjeka na intervalu $[-30\pi, 30\pi]$ je $30 \cdot 2 - 1 = 59$.

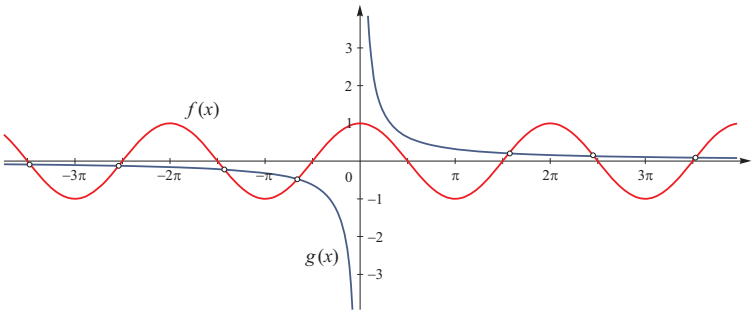
Pogledajmo još da li se funkcije f i g sijeku na intervalu $[31\pi, 31.83\pi]$.



Na svakom od intevral $[-31.83\pi, -31\pi]$, $[31\pi, 31.83\pi]$ ima jos po dva rješenja, tj. još ukupno 4, te je broj ukupnih rješenja:

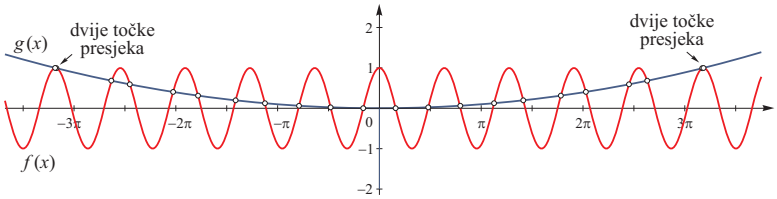
$$59 + 4 = 63.$$

2) $\underbrace{\cos x}_{f(x)} = \underbrace{\dfrac{1}{x}}_{g(x)}$;



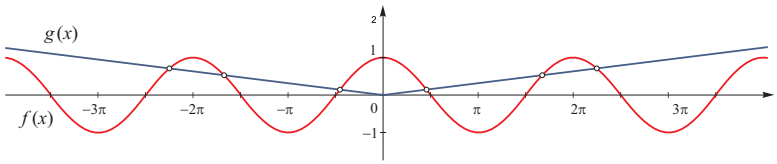
Ima beskonačno mnogo rješenja.

3) $\underbrace{\cos(\pi x)}_{f(x)} = \underbrace{\dfrac{x^2}{100}}_{g(x)}$;



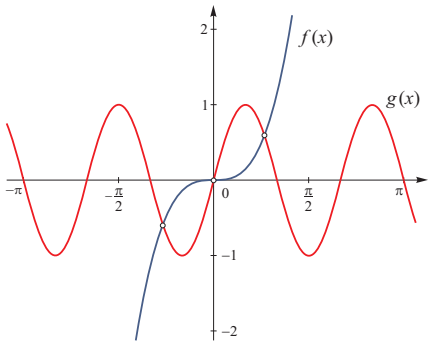
Ima 22 rješenja.

4) $\underbrace{\cos x}_{f(x)} = \underbrace{\dfrac{|x|}{10}}_{g(x)}$;



Ima 6 rješenja.

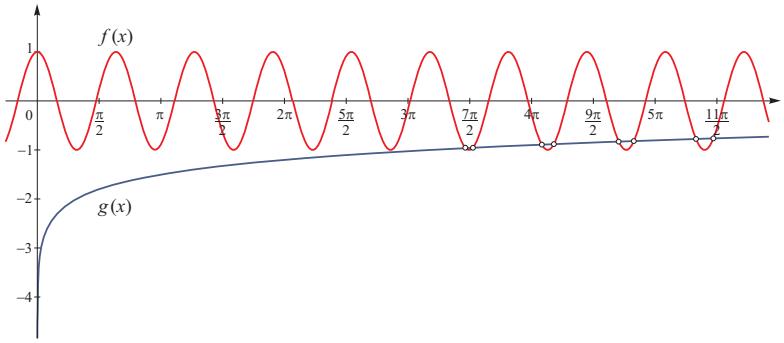
5) $\underbrace{x^3}_{f(x)} = \underbrace{\sin(3x)}_{g(x)}$;



Ima 3 rješenja.

6) $\underbrace{\cos(\pi x)}_{f(x)} = \underbrace{\log \frac{x}{100}}_{g(x)}$

Zbog $|\cos \pi x| \leqslant 1$ funkcije f i g se sijeku samo za $\log \frac{x}{100} \geqslant -1$, $\frac{x}{100} \geqslant \frac{1}{10}$ tj. $x \geqslant 10$, odnosno $\log \frac{x}{100} \leqslant 1$, tj. $x \leqslant 1000$.
Sve točke presjeka su na intervalu $[10, 1000]$, odnosno $[3.183\pi, 318.31\pi]$.



Period funkcije $f(x)$ je $P = \frac{2\pi}{\pi} = 2$. Na intervalu $[10, 1000]$ ima $\frac{1000 - 10}{2} = 495$ perioda. Na svakom periodu siječe funkciju $g(x)$ u dvije točke te je ukupan broj presjeka:

$$495 \cdot 2 = 990.$$