Zadatak 22.

1) $\begin{cases} \sin^2 x + \sin^2 y = \dfrac{1}{4} \\ x - y = \dfrac{\pi}{6} \end{cases}$

2) $\begin{cases} \cos^2 x + \cos^2 y = \dfrac{7}{4} \\ x - y = \dfrac{5\pi}{6} \end{cases}$

3) $\begin{cases} \sin \pi x - \sin \pi y = \sqrt{3} \\ y - x = 3 \end{cases}$

4) $\begin{cases} \cos \pi x - \cos \pi y = -\sqrt{3} \\ x - y = 1 \end{cases}$

Rješenje. **1)**

$$\sin^2 x + \sin^2 y = \frac{1}{4}$$
$$x - y = \frac{\pi}{6} \implies x = y + \frac{\pi}{6}$$

$$\sin^2\left(y + \frac{\pi}{6}\right) + \sin^2 y = \frac{1}{4}$$
$$\left(\sin y \cos\frac{\pi}{6} + \cos y \sin\frac{\pi}{6}\right)^2 + \sin^2 y = \frac{1}{4}$$
$$\left(\frac{\sqrt{3}}{2}\sin y + \frac{1}{2}\cos y\right)^2 + \sin^2 y = \frac{1}{4}$$
$$\frac{3}{4}\sin^2 y + \frac{\sqrt{3}}{2}\sin y \cos y + \frac{1}{4}\cos^2 y + \sin^2 y = \frac{1}{4}(\sin^2 y + \cos^2 y)$$
$$\frac{3}{4}\sin^2 y + \frac{\sqrt{3}}{2}\sin y \cos y + \frac{1}{4}\cos^2 y + \sin^2 y = \frac{1}{4}(\sin^2 y + \cos^2 y) \quad / \cdot 4$$
$$3\sin^2 y + 2\sqrt{3}\sin y \cos y + \cos^2 y + 4\sin^2 y - \sin^2 y - \cos^2 y = 0$$
$$6\sin^2 y + 2\sqrt{3}\sin y \cos y = 0$$
$$2\sin y(\sin y + 2\sqrt{3}\cos y) = 0$$

$1^\circ \quad 2\sin y = 0, \quad \sin y = 0 \implies y_1 = k\pi, \quad x_1 = y_1 + \dfrac{\pi}{6} = \dfrac{\pi}{6} + k\pi, \quad k \in \mathbf{Z}$

$2^\circ \quad \sin y + 2\sqrt{3}\cos y = 0 \quad / : \sqrt{3}\cos y$

$$\frac{3}{\sqrt{3}}\operatorname{tg} y + 1 = 0$$

$$\operatorname{tg} y = -\frac{\sqrt{3}}{3} \implies y_2 = -\frac{\pi}{6} + k\pi, \quad x_2 = y_2 + \frac{\pi}{6} = -\frac{\pi}{6} + k\pi + \frac{\pi}{6} = k\pi, \quad k \in \mathbf{Z}$$

$$\implies (x_1, y_1) = \left(\frac{\pi}{6} + k\pi, k\pi\right), \quad (x_2, y_2) = \left(k\pi, -\frac{\pi}{6} + k\pi\right);$$

2)

$$\cos^2 x + \cos^2 y = \frac{7}{4}$$
$$x - y = \frac{5\pi}{6} \implies x = y + \frac{5\pi}{6}$$

$$\cos^2\left(y + \frac{5\pi}{6}\right) + \cos^2 y = \frac{7}{4}$$
$$\left(\cos y \cos\frac{5\pi}{6} - \sin y \sin\frac{5\pi}{6}\right)^2 + \cos^2 y = \frac{7}{4}$$
$$\left(-\frac{\sqrt{3}}{2}\cos y - \frac{1}{2}\sin y\right)^2 + \cos^2 y = \frac{7}{4}$$
$$\frac{3}{4}\cos^2 y + \frac{\sqrt{3}}{2}\sin y\cos y + \frac{1}{4}\sin^2 y + \cos^2 y = \frac{7}{4} \quad / \cdot 4$$
$$3\cos^2 y + 2\sqrt{3}\sin y\cos y + \sin^2 y + 4\cos^2 y - 7\sin^2 y - 7\cos^2 y = 0$$
$$2\sqrt{3}\sin y\cos y - 6\sin^2 y = 0$$
$$2\sin y(\sqrt{3}\cos y - 3\sin y) = 0$$

$1^\circ \quad 2\sin y = 0, \quad \sin y = 0 \implies y_1 = k\pi, \quad x_1 = y_1 + \frac{5\pi}{6} = \frac{5\pi}{6} + k\pi, \quad k \in \mathbf{Z}$

$2^\circ \quad \sqrt{3}\cos y - 3\sin y = 0 \quad / : \sqrt{3}\cos y$

$$1 - \sqrt{3}\,\mathrm{tg}\, y = 0$$
$$\mathrm{tg}\, y = \frac{\sqrt{3}}{3} \implies y_2 = -\frac{5\pi}{6} + k\pi, \quad x_2 = y_2 + \frac{5\pi}{6} = -\frac{5\pi}{6} + k\pi + \frac{5\pi}{6} = k\pi, \quad k \in \mathbf{Z}$$

$$\implies (x_1, y_1) = \left(\frac{5\pi}{6} + k\pi, k\pi\right), \quad (x_2, y_2) = \left(k\pi, -\frac{5\pi}{6} + k\pi\right);$$

3)

$$\sin \pi x - \sin \pi y = \sqrt{3}$$
$$y - x = 3 \implies y = x + 3$$

$$\sin \pi x - \sin \pi(x+3) = \sqrt{3}$$
$$\sin \pi x - \sin(\pi x + 3\pi) = \sqrt{3}$$
$$\sin \pi x - (\sin \pi x \cos 3\pi + \cos \pi x \sin 3\pi) = \sqrt{3}$$
$$\sin \pi x + \sin \pi x = \sqrt{3}$$
$$2\sin \pi x = \sqrt{3} \quad / : 2$$
$$\sin \pi x = \frac{\sqrt{3}}{2}$$

$1^\circ \quad \pi x_1 = \frac{\pi}{3} + 2k\pi, \quad x_1 = \frac{1}{3} + 2k, \quad y_1 = x_1 + 3 = \frac{10}{3} + 2k \quad k \in \mathbf{Z}$

$2^\circ \quad \pi x_2 = \frac{2\pi}{3} + 2k\pi, \quad x_2 = \frac{2}{3} + 2k, \quad y_2 = x_2 + 3 = \frac{11}{3} + 2k \quad k \in \mathbf{Z}$

$$\implies (x_1, y_1) = \left(\frac{1}{3} + 2k, \frac{10}{3} + 2k\right), \quad (x_2, y_2) = \left(\frac{2}{3} + 2k, \frac{11}{3} + 2k\right);$$

4)

$\cos \pi x - \cos \pi y = -\sqrt{3}$

$x - y = 1 \implies x = y + 1$

$\cos \pi(y+1) - \cos \pi y = -\sqrt{3}$

$\cos \pi y \cos \pi - \sin \pi y \sin \pi - \cos \pi y = -\sqrt{3}$

$-\cos \pi y - \cos \pi y = -\sqrt{3}$

$2 \cos \pi y = \sqrt{3}$

$\cos \pi y = \dfrac{\sqrt{3}}{2}$

$1^\circ \quad \pi y_1 = \dfrac{\pi}{6} + 2k\pi, \quad y_1 = \dfrac{1}{6} + 2k, \quad x_1 = y_1 + 1 = \dfrac{7}{6} + 2k \quad k \in \mathbf{Z}$

$2^\circ \quad \pi y_2 = -\dfrac{\pi}{6} + 2k\pi, \quad y_2 = -\dfrac{1}{6} + 2k, \quad x_2 = y_2 + 1 = \dfrac{5}{6} + 2k \quad k \in \mathbf{Z}$

$\implies (x_1, y_1) = \left(\dfrac{7}{6} + 2k, \dfrac{1}{6} + 2k\right), \quad (x_2, y_2) = \left(\dfrac{5}{6} + 2k, -\dfrac{1}{6} + 2k\right);$