Zadatak 23.

1) $\begin{cases} \operatorname{tg} x + \operatorname{tg} y = 1 \\ x - y = \dfrac{\pi}{4} \end{cases}$

2) $\begin{cases} \operatorname{tg} x \cdot \operatorname{tg} y = 3 \\ x + y = \dfrac{2\pi}{3} \end{cases}$

3) $\begin{cases} x + y = \frac{\pi}{4} \\ \operatorname{tg} x - \operatorname{tg} y = 2 \end{cases}$

4) $\begin{cases} x + y = \frac{3\pi}{4} \\ \operatorname{tg} y - \operatorname{tg} x = 2 \end{cases}$

Rješenje. **1)**

$$\operatorname{tg} x + \operatorname{tg} y = 1$$

$$x - y = \frac{\pi}{4} \implies x = y + \frac{\pi}{4}$$

$$\operatorname{tg}\left(y + \frac{\pi}{4}\right) + \operatorname{tg} y = 1$$

$$\frac{\operatorname{tg} y + \operatorname{tg} \frac{\pi}{4}}{1 - \operatorname{tg} y \cdot \operatorname{tg} \frac{\pi}{4}} + \operatorname{tg} y = 1$$

$$\frac{\operatorname{tg} y + 1}{1 - \operatorname{tg} y} + \operatorname{tg} y - 1 = 0 \quad / \cdot (1 - \operatorname{tg} y)$$

$$\operatorname{tg} y + 1 - (1 - \operatorname{tg} y)^2 = 0$$

$$\operatorname{tg} y + 1 - 1 + 2 \operatorname{tg} y - \operatorname{tg}^2 y = 0$$

$$-\operatorname{tg}^2 y + 3 \operatorname{tg} y = 0$$

$$\operatorname{tg} y(-\operatorname{tg} y + 3) = 0$$

$1^\circ \quad \operatorname{tg} y = 0 \implies y_1 = k\pi, \quad x_1 = \dfrac{\pi}{4} + k\pi, \quad k \in \mathbf{Z}$

$2^\circ \quad -\operatorname{tg} y + 3 = 0,\ \operatorname{tg} y = 3 \implies y_2 = \operatorname{arc\,tg} 3 + k\pi,\ x_2 = \dfrac{\pi}{4} + \operatorname{arc\,tg} 3 + k\pi,\ k \in \mathbf{Z}$

$$\implies (x_1, y_1) = \left(\frac{\pi}{4} + k\pi,\ k\pi\right), \quad (x_2, y_2) = \left(\frac{\pi}{4} + \operatorname{arc\,tg} 3 + k\pi,\ \operatorname{arc\,tg} 3 + k\pi\right);$$

2)

$$\operatorname{tg} x \cdot \operatorname{tg} y = 3$$

$$x + y = \frac{2\pi}{3} \implies y = \frac{2\pi}{3} - x$$

$$\operatorname{tg} x \cdot \operatorname{tg}\left(\frac{2\pi}{3} - x\right) = 3$$

$$\operatorname{tg} x \cdot \frac{\operatorname{tg} \frac{2\pi}{3} - \operatorname{tg} x}{1 + \operatorname{tg} \frac{2\pi}{3} \operatorname{tg} x} = 3$$

$$\operatorname{tg} x \cdot \frac{-\sqrt{3}-\operatorname{tg} x}{1-\sqrt{3}\operatorname{tg} x} = 3$$

$$\frac{-\sqrt{3}\operatorname{tg} x - \operatorname{tg}^2 x}{1-\sqrt{3}\operatorname{tg} x} = 3 \quad / \cdot (1-\sqrt{3}\operatorname{tg} x)$$

$$-\sqrt{3}\operatorname{tg} x - \operatorname{tg}^2 x = 3 - 3\sqrt{3}\operatorname{tg} x$$

$$-\operatorname{tg}^2 x + 2\sqrt{3}\operatorname{tg} x - 3 = 0 \quad / \cdot (-1)$$

$$\operatorname{tg}^2 x - 2\sqrt{3}\operatorname{tg} x + 3 = 0$$

$$(\operatorname{tg} x)_{1,2} = \frac{2\sqrt{3} \pm \sqrt{12-12}}{2} = \sqrt{3}$$

$$x = \frac{\pi}{3} + k\pi, \quad y = \frac{\pi}{3} + k\pi, \quad k \in \mathbf{Z};$$

3) $x + y = \frac{\pi}{4} \implies y = \frac{\pi}{4} - x$

$\operatorname{tg} x - \operatorname{tg} y = 2$

$$\operatorname{tg} x - \operatorname{tg}\left(\frac{\pi}{4} - x\right) = 2$$

$$\operatorname{tg} x - \frac{\operatorname{tg}\frac{\pi}{4} - \operatorname{tg} x}{1 + \operatorname{tg}\frac{\pi}{4}\operatorname{tg} x} = 2$$

$$\operatorname{tg} x - \frac{1-\operatorname{tg} x}{1+\operatorname{tg} x} = 2 \quad / \cdot (1+\operatorname{tg} x)$$

$$\operatorname{tg} x + \operatorname{tg}^2 x - 1 + \operatorname{tg} x = 2 + 2\operatorname{tg} x$$

$$\operatorname{tg}^2 x = 3 \quad /\sqrt{\ }$$

$$\operatorname{tg} x = \pm\sqrt{3}$$

$$1^\circ \quad \operatorname{tg} x = \sqrt{3} \implies x_1 = \frac{\pi}{3} + k\pi, \quad y_1 = -\frac{\pi}{12} + k\pi, \quad k \in \mathbf{Z}$$

$$2^\circ \quad \operatorname{tg} y - \sqrt{3} \implies x_2 = -\frac{\pi}{3} + k\pi, \quad y_2 = \frac{7\pi}{12} + k\pi, \quad k \in \mathbf{Z}$$

$$\implies (x_1, y_1) = \left(\frac{\pi}{3} + k\pi, -\frac{\pi}{12} + k\pi\right), \quad (x_2, y_2) = \left(-\frac{\pi}{3} + k\pi, \frac{7\pi}{12} + k\pi\right);$$

4) $x + y = \frac{3\pi}{4} \implies y = \frac{3\pi}{4} - x$

$\operatorname{tg} y - \operatorname{tg} x = 2$

$$\operatorname{tg}\left(\frac{3\pi}{4} - x\right) - \operatorname{tg} x = 2$$

$$\frac{\operatorname{tg}\frac{3\pi}{4}-\operatorname{tg}x}{1+\operatorname{tg}\frac{3\pi}{4}\operatorname{tg}x}-\operatorname{tg}x=2$$

$$\frac{-1-\operatorname{tg}x}{1-\operatorname{tg}x}-\operatorname{tg}x=2 \quad / \cdot (1-\operatorname{tg}x)$$

$$-1-\operatorname{tg}x-\operatorname{tg}x+\operatorname{tg}^2 x=2-2\operatorname{tg}x$$

$$\operatorname{tg}^2 x=3 \quad / \sqrt{\ }$$

$$\operatorname{tg}x=\pm\sqrt{3}$$

$$1^\circ \quad \operatorname{tg}x=\sqrt{3} \implies x_1=\frac{\pi}{3}+k\pi, \quad y_1=\frac{3\pi}{4}-x_1=\frac{5\pi}{12}+k\pi, \quad k\in\mathbf{Z}$$

$$2^\circ \quad \operatorname{tg}y-\sqrt{3} \implies x_2=-\frac{\pi}{3}+k\pi, \quad y_2=\frac{3\pi}{4}-x_2=\frac{13\pi}{12}+k\pi, \quad k\in\mathbf{Z}$$

$$\implies (x_1,y_1)=\left(\frac{\pi}{3}+k\pi,\ \frac{5\pi}{12}+k\pi\right), \quad (x_2,y_2)=\left(-\frac{\pi}{3}+k\pi,\ \frac{13\pi}{12}+k\pi\right);$$