Zadatak 24.

1) $\begin{cases} |x|+|y|=3 \\ \sin\dfrac{\pi x^2}{2}=1 \end{cases}$

2) $\begin{cases} 2^{\cos x}+2^{\frac{1}{\cos y}}=5 \\ 2^{\cos x}\cdot 2^{\frac{1}{\cos y}}=4 \end{cases}$

Rješenje. **1)**

$$|x|+|y|=3 \implies |x|,\ |y|\leqslant 3$$

$$\sin\frac{\pi x^2}{2}=1$$

$$\frac{\pi x^2}{2}=\frac{\pi}{2}+2k\pi$$

$$x^2=1+4k$$

$$x_{1,2}=\pm\sqrt{1+4k}$$

$$|x|\leqslant 3 \quad \text{i} \quad 1+4k\geqslant 0, \quad k\geqslant -\frac{1}{4}$$

$\underline{k=0}$

$$x_{1,2}=\pm 1, \quad 1+|y|=3, \quad |y|=2,$$

$$y_{1,2}=\pm 2 \implies (\pm 1,\pm 2) \quad \text{4 rješenja;}$$

$\underline{k=1}$

$$x_{3,4}=\pm\sqrt{5}, \quad \sqrt{5}+|y|=3, \quad |y|=3-\sqrt{5}$$

$$y_{3,4}=\pm(3-\sqrt{5}) \implies (\pm\sqrt{5},\pm(3-\sqrt{5})) \quad \text{4 rješenja;}$$

$\underline{k=2}$

$$x_{5,6}=\pm 3, \quad 3+|y|=3,$$

$$y=0, \implies (\pm 3,0) \quad \text{2 rješenja;}$$

Zadatak ima ukupno 10 rješenja.

2)

$$2^{\cos x}+2^{\frac{1}{\cos y}}=5 \qquad (1)$$

$$2^{\cos x}\cdot 2^{\frac{1}{\cos y}}=4 \qquad (2)$$

iz (2)

$$2^{\cos x+\frac{1}{\cos y}}=2^2$$

$$\cos x+\frac{1}{\cos y}=2 \quad /\cdot\cos y$$

$$\cos x\cos y+1=2\cos y$$

$$\cos y(\cos x-2)=1 \quad /:(\cos x-2)$$

$$\cos y = \frac{1}{2-\cos x} \qquad (3)$$

Uvrstimo (3) u (1):

$$2^{\cos x} + 2^{\frac{1}{\frac{1}{2-\cos x}}} = 5\&$$

$$2^{\cos x} + 2^{2-\cos x} = 5$$

$$2^{\cos x} = t$$

$$t + 2^2 \cdot t^{-1} = 5$$

$$t + \frac{4}{t} = 5 \quad / \cdot t$$

$$t^2 - 5t + 4 = 0$$

$$t_{1,2} = \frac{5 \pm \sqrt{25-16}}{2} = \frac{5 \pm 3}{2} \implies t_1 = 4, \quad t_2 = 1$$

$\underline{t = 4}$

$$2^{\cos x} = 4$$

$$2^{\cos x} = 2^2$$

$\cos x = 2$ nije rješenje

$\underline{t = 1}$

$$2^{\cos x} = 1$$

$$2^{\cos x} = 2^0$$

$$\cos x = 0 \implies \underline{x = \frac{\pi}{2} + 2k\pi}, \quad k \in \mathbf{Z}$$

uvrstimo dobiveno u (3) pa imamo:

$$\cos y = \frac{1}{2-0} = \frac{1}{2} \implies \underline{y = \pm\frac{\pi}{3} + 2k\pi}, \quad k \in \mathbf{Z}$$