Zadatak 4.

1) $2\sin(\frac{x}{2}-\frac{\pi}{4}) \geqslant -1$;

2) $2\cos(3x-\frac{\pi}{3}) < \sqrt{3}$;

3) $\sqrt{3}\cdot \mathrm{tg}(\frac{x}{3}+\frac{\pi}{6})-1 \geqslant 0$;

4) $\mathrm{ctg}(\frac{3\pi}{2}+\frac{x}{2})-1 \leqslant 0$.

Rješenje.

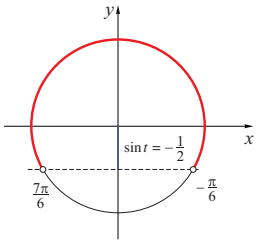
1) $2\sin\left(\frac{x}{2}-\frac{\pi}{4}\right) \geqslant -1$;

$\sin\left(\frac{x}{2}-\frac{\pi}{4}\right) \geqslant -\frac{1}{2}$;

$-\frac{\pi}{6}+2k\pi \leqslant \frac{x}{2}-\frac{\pi}{4} \leqslant \frac{7\pi}{6}+2k\pi,\ k\in\mathbf{Z}$;

$\frac{\pi}{12}+2k\pi \leqslant \frac{x}{2} \leqslant \frac{17\pi}{12}+2k\pi,\ k\in\mathbf{Z}$;

$\frac{\pi}{6}+4k\pi \leqslant x \leqslant \frac{17\pi}{6}+4k\pi,\ k\in\mathbf{Z}$;



2) $2\cos\left(3x-\frac{\pi}{3}\right) < \sqrt{3}$,

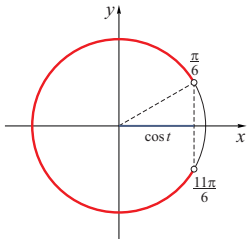
$\cos\left(3x-\frac{\pi}{3}\right) < \frac{\sqrt{3}}{2}$;

$\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$;

$\frac{\pi}{6}+2k\pi < 3x-\frac{\pi}{3} < \frac{11\pi}{6}+2k\pi,\ k\in\mathbf{Z}$;

$\frac{\pi}{2}+2k\pi < 3x < \frac{13\pi}{6}+2k\pi,\ k\in\mathbf{Z}$;

$\frac{\pi}{6}+\frac{2k\pi}{3} < x < \frac{13\pi}{18}+\frac{2k\pi}{3},\ k\in\mathbf{Z}$;



3) $\sqrt{3}\cdot \operatorname{tg}\left(\frac{x}{3}+\frac{\pi}{6}\right)-1\geqslant 0\,;$

$\sqrt{3}\cdot \operatorname{tg}\left(\frac{x}{3}+\frac{\pi}{6}\right)\geqslant 1\,;$

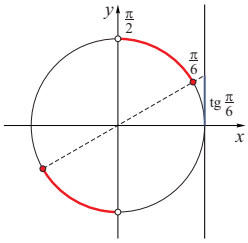
$\operatorname{tg}\left(\frac{x}{3}+\frac{\pi}{6}\right)\geqslant \frac{\sqrt{3}}{3}\,;$

$\operatorname{tg}\frac{\pi}{6}=\frac{\sqrt{3}}{3}\,;$

$\frac{\pi}{6}+k\pi\leqslant \frac{x}{3}+\frac{\pi}{6}<\frac{\pi}{2}+k\pi\,;$

$k\pi\leqslant \frac{x}{3}<\frac{\pi}{3}+k\pi\,;$

$k\cdot 3\pi\leqslant x<(3k+1)\cdot\pi\,,\ k\in\mathbf{Z}\,.$



4) $\operatorname{ctg}\left(\frac{3\pi}{2}+\frac{x}{2}\right)-1\leqslant 0\,,$

$\operatorname{ctg}\left(\frac{3\pi}{2}+\frac{x}{2}\right)\leqslant 1\,;$

$\operatorname{ctg}\frac{\pi}{4}=1\,;$

$\frac{\pi}{4}+k\pi\leqslant \frac{3\pi}{2}+\frac{x}{2}<\pi+k\pi\,;$

$-\frac{5\pi}{4}+k\pi\leqslant \frac{x}{2}<-\frac{\pi}{2}+k\pi\,;$

$-\frac{\pi}{4}+k\pi\leqslant \frac{x}{2}<\frac{\pi}{2}+k\pi\,;$

$-\frac{\pi}{2}+2k\pi\leqslant x<(2k+1)\cdot\pi\,,\ k\in\mathbf{Z}\,.$