Zadatak 6.

1) $\sin x + \cos x + \sin 2x > 1\,;$

2) $\cos x - \sin x + \sin x \cos x > \frac{1}{2}\,;$

3) $x \cdot \sin x + 1 > x + \sin x\,;$

4) $\operatorname{ctg}^3 x + \operatorname{ctg}^2 x - \operatorname{ctg} x - 1 < 0\,.$

Rješenje. **1)**

$$\sin x + \cos x + \sin 2x > 1$$

$$\sin x + \sin\left(\frac{\pi}{2} - x\right) + 2\sin x \sin\left(\frac{\pi}{2} - x\right) > 1$$

$$2\sin\frac{x + \frac{\pi}{2} - x}{2}\cos\frac{x - \frac{\pi}{2} + x}{2} + 2\cdot\frac{1}{2}\left[\cos\left(x - \frac{\pi}{2} + x\right) - \cos\left(x + \frac{\pi}{2} - x\right)\right] > 1$$

$$\sqrt{2}\cos\left(x - \frac{\pi}{4}\right) + \cos 2\left(x - \frac{\pi}{4}\right) > 1$$

$$\sqrt{2}\cos\left(x - \frac{\pi}{4}\right) + 2\cos^2\left(x - \frac{\pi}{4}\right) - 1 > 1$$

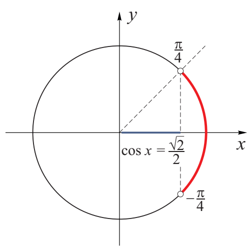
$$\sqrt{2}\cos\left(x - \frac{\pi}{4}\right) + 2\cos^2\left(x - \frac{\pi}{4}\right) - 2 > 0$$

$$t = \cos\left(x - \frac{\pi}{4}\right), \quad |t| \leqslant 1$$

$$2t^2 + \sqrt{2}t - 2 > 0$$

$$t_{1,2} = \frac{-\sqrt{2} \pm \sqrt{2 + 16}}{4} = \frac{-\sqrt{2} \pm 3\sqrt{2}}{4}$$

$$t_1 = -\sqrt{2} \text{ nije rješenje}, \quad t_2 = \frac{\sqrt{2}}{2} \implies \cos\left(x - \frac{\pi}{4}\right) \geqslant \frac{\sqrt{2}}{2}$$



$$-\frac{\pi}{4} + 2k\pi \leqslant x - \frac{\pi}{4} \leqslant \frac{\pi}{4} + 2k\pi$$

$$2k\pi \leqslant x \leqslant \frac{\pi}{2} + 2k\pi, \quad k \in \mathbf{Z}$$

2)

$$\cos x - \sin x + \sin x \cos x > \frac{1}{2}$$

$$\cos x - \sin x + \sin x \cos x - \frac{1}{2} > 0.2$$

$$2(\cos x - \sin x) + 2\sin x \cos x - 1 > 0$$

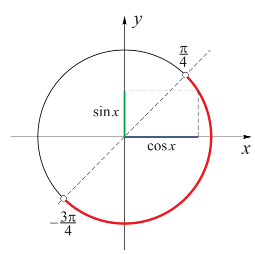
$$2(\cos x - \sin x) + 2\sin x \cos x - \sin^2 x - \cos^2 x > 0$$

$$2(\cos x - \sin x) - (\cos x - \sin x)^2 > 0$$

$$(\cos x - \sin x)\underbrace{(2 - \cos x + \sin x)}_{>0} > 0$$

$$\implies \cos x - \sin x \geqslant 0 \quad \cos x \geqslant \sin x$$

$$-\frac{3\pi}{4} + k\cdot 2\pi < x < \frac{\pi}{4} + k\cdot 2\pi,\ k \in \mathbf{Z}\,;$$



3)

$$
\begin{aligned}
&x \cdot \sin x + 1 > x + \sin x\\
&x \cdot \sin x - x + 1 - \sin x > 0\\
&x(\sin x - 1) - (\sin x - 1) > 0\\
&\underbrace{(\sin x - 1)}_{\leqslant 0}(x - 1) > 0
\end{aligned}
$$

Mora vrijediti

$$x - 1 < 0,\ \ x < 1 \quad \text{i} \quad \sin x \neq 1, \quad x \neq \frac{\pi}{2} - 2k\pi, \quad k = 1, 2, 3\ldots$$

$$\Longrightarrow\ x \in \langle -\infty, 1\rangle \setminus \left\{\frac{\pi}{2} - 2k\pi,\ \ k = 1, 2, 3, \ldots\right\}$$

4)

$$\operatorname{ctg}^3 x + \operatorname{ctg}^2 x - \operatorname{ctg} x - 1 < 0$$

$$\operatorname{ctg}^2 x(\operatorname{ctg} x + 1) - (\operatorname{ctg} x + 1) < 0$$

$$(\operatorname{ctg} x + 1)(\operatorname{ctg}^2 x - 1) < 0$$

1° $\operatorname{ctg} x + 1 < 0$

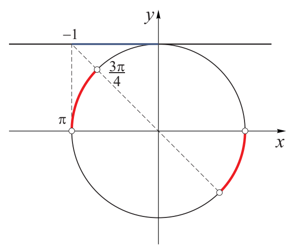
$\operatorname{ctg}^2 x - 1 > 0$

$\operatorname{ctg} x < -1$

$\operatorname{ctg}^2 x > 1$

$\Longrightarrow \operatorname{ctg} x < -1$

$$\frac{3\pi}{4} + k\pi < x < \pi + k\pi$$



2° $\operatorname{ctg} x + 1 > 0$

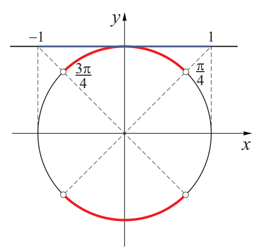
$\operatorname{ctg}^2 x - 1 < 0$

$\operatorname{ctg} x > -1$

$\operatorname{ctg}^2 x < 1$

$\Longrightarrow \operatorname{ctg} x \in \langle -1, 1\rangle$

$$\frac{\pi}{4} + k\pi < x < \frac{3\pi}{4} + k\pi$$



$$\frac{\pi}{4} + k \cdot \pi < x < \frac{3\pi}{4} + k \cdot \pi \ \text{ ili } \ \frac{3\pi}{4} + k \cdot \pi < x < (k+1)\pi,\ k \in \mathbf{Z}.$$