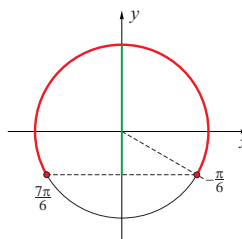


- Zadatak 8.**
- 1) $\sin x \cdot \cos x \geq -\frac{1}{4}$;
 - 2) $\cos^2 x - \sin^2 x < -\frac{\sqrt{3}}{2}$;
 - 3) $\cos^2 \frac{x}{3} \leq \sin^2 \frac{x}{3} - \frac{1}{2}$;
 - 4) $\sin\left(\frac{\pi}{3} - 2x\right) \cos\left(\frac{\pi}{3} - 2x\right) \geq -\frac{\sqrt{3}}{4}$;
 - 5) $\sin 2x + \sqrt{3} \cos 2x \geq 1$;
 - 6) $\sqrt{3} \sin \frac{x}{2} - \cos \frac{x}{2} \leq \sqrt{3}$;
 - 7) $2 \sin^2\left(x + \frac{\pi}{4}\right) + \sqrt{3} \cos 2x > 0$;
 - 8) $\sin x \cdot \sin 2x - \cos x \cdot \cos 2x > \sin 6x$.

Rješenje.

1)

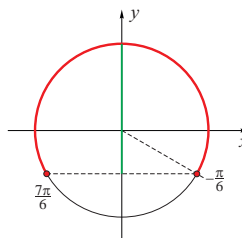
$$\begin{aligned} \sin x \cdot \cos x &\geq -\frac{1}{4} \\ \frac{1}{2} \cdot 2 \sin x \cdot \cos x &\geq -\frac{1}{4} \\ \sin 2x &\geq -\frac{1}{2} \end{aligned}$$



$$\begin{aligned} -\frac{\pi}{6} + 2k\pi &\leq 2x \leq \frac{7\pi}{6} + 2k\pi \\ -\frac{\pi}{12} + k\pi &\leq x \leq \frac{7\pi}{12} + k\pi, \quad k \in \mathbf{Z} \end{aligned}$$

2)

$$\begin{aligned} \cos^2 x - \sin^2 x &< -\frac{\sqrt{3}}{2} \\ \cos 2x &< -\frac{\sqrt{3}}{2} \quad \left(\cos \frac{5\pi}{6} = -\frac{\sqrt{3}}{2}\right) \end{aligned}$$



$$\frac{5\pi}{6} + 2k\pi < 2x < \frac{7\pi}{6} + 2k\pi$$

$$\frac{5\pi}{12} + k\pi < x < \frac{7\pi}{12} + k\pi, \quad k \in \mathbf{Z}$$

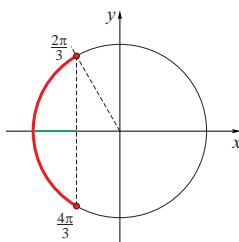
3)

$$\cos^2 \frac{x}{3} \leq \sin^2 \frac{x}{3} - \frac{1}{2}$$

$$\cos^2 \frac{x}{3} - \sin^2 \frac{x}{3} \leq -\frac{1}{2}$$

$$\cos\left(2 \cdot \frac{x}{3}\right) \leq -\frac{1}{2}$$

$$\cos \frac{2}{3}x \leq -\frac{1}{2} \quad \left(\cos \frac{2\pi}{3} = -\frac{1}{2}\right)$$



$$\frac{2\pi}{3} + 2k\pi \leq \frac{2}{3}x \leq \frac{4\pi}{3} + 2k\pi$$

$$\pi + 3k\pi \leq x \leq 2\pi + 3k\pi$$

$$(3k+1)\pi \leq x \leq (3k+2)\pi$$

4)

$$\sin\left(\frac{\pi}{3} - 2x\right) \cos\left(\frac{\pi}{3} - 2x\right) \geq -\frac{\sqrt{3}}{4} \quad / \cdot 2$$

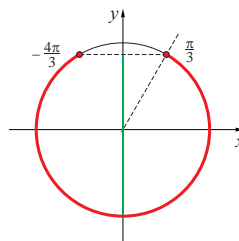
$$2 \sin\left(\frac{\pi}{3} - 2x\right) \cos\left(\frac{\pi}{3} - 2x\right) \geq -\frac{\sqrt{3}}{2}$$

$$\sin\left[2 \cdot \left(\frac{\pi}{3} - 2x\right)\right] \geq -\frac{\sqrt{3}}{2}$$

$$\sin\left(\frac{2\pi}{3} - 4x\right) \geq -\frac{\sqrt{3}}{2}$$

$$-\sin\left(4x - \frac{2\pi}{3}\right) \geq -\frac{\sqrt{3}}{2}$$

$$\sin\left(4x - \frac{2\pi}{3}\right) \leq \frac{\sqrt{3}}{2} \quad \left(\sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}\right)$$



$$-\frac{4\pi}{3} + 2k\pi \leq 4x - \frac{2\pi}{3} \leq \frac{\pi}{3} + 2k\pi$$

$$-\frac{2\pi}{3} + 2k\pi \leq 4x \leq \pi + 2k\pi$$

$$-\frac{\pi}{6} + \frac{k\pi}{2} \leq x \leq \frac{\pi}{4} + \frac{k\pi}{2}$$

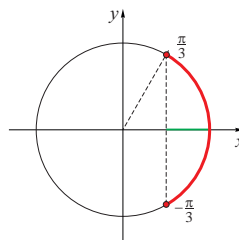
5)

$$\sin 2x + \sqrt{3} \cos 2x \geq 1 \quad / : 2$$

$$\frac{1}{2} \sin 2x + \frac{\sqrt{3}}{2} \cos 2x \geq \frac{1}{2}$$

$$\sin \frac{\pi}{6} \sin 2x + \cos \frac{\pi}{6} \cos 2x \geq \frac{1}{2}$$

$$\cos\left(2x - \frac{\pi}{6}\right) \geq \frac{1}{2}$$



$$-\frac{\pi}{3} + 2k\pi \leq 2x - \frac{\pi}{6} \leq \frac{\pi}{3} + 2k\pi$$

$$-\frac{\pi}{6} + 2k\pi \leq 2x \leq \frac{\pi}{2} + 2k\pi$$

$$-\frac{\pi}{12} + k\pi \leq x \leq \frac{\pi}{4} + k\pi$$

6)

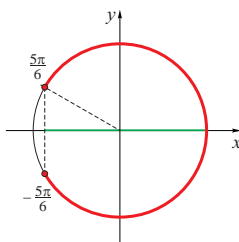
$$\sqrt{3} \sin \frac{x}{2} - \cos \frac{x}{2} \leq \sqrt{3} \quad / : 2$$

$$\frac{\sqrt{3}}{2} \sin \frac{x}{2} - \frac{1}{2} \cos \frac{x}{2} \leq \frac{\sqrt{3}}{2}$$

$$\sin \frac{\pi}{3} \sin \frac{x}{2} - \cos \frac{\pi}{3} \cos \frac{x}{2} \leq \frac{\sqrt{3}}{2}$$

$$-\cos \left(\frac{x}{2} + \frac{\pi}{3} \right) \leq \frac{\sqrt{3}}{2}$$

$$\cos \left(\frac{x}{2} + \frac{\pi}{3} \right) \geq -\frac{\sqrt{3}}{2}, \quad \left(\cos \frac{5\pi}{6} = -\frac{\sqrt{3}}{2} \right)$$



$$-\frac{5\pi}{6} + 2k\pi \leq \frac{x}{2} + \frac{\pi}{3} \leq \frac{5\pi}{6} + 2k\pi$$

$$-\frac{7\pi}{6} + 2k\pi \leq \frac{x}{2} \leq \frac{\pi}{2} + 2k\pi$$

$$-\frac{7\pi}{3} + 4k\pi \leq x \leq \pi + 4k\pi$$

$$-\frac{7\pi}{3} + 4k\pi \leq x \leq (4k+1)\pi$$

7)

$$2 \sin^2 \left(x + \frac{\pi}{4} \right) + \sqrt{3} \cos 2x > 0$$

$$2 \left(\sin x \cos \frac{\pi}{4} + \cos x \sin \frac{\pi}{4} \right)^2 + \sqrt{3} \cos 2x > 0$$

$$2 \cdot \left(\frac{\sqrt{2}}{2} \right)^2 (\sin x + \cos x)^2 + \sqrt{3} \cos 2x > 0$$

$$\sin^2 x + 2 \sin x \cos x + \cos^2 x + \sqrt{3} \cos 2x > 0$$

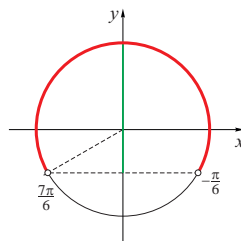
$$1 + \sin 2x + \sqrt{3} \cos 2x > 0$$

$$\sin 2x + \sqrt{3} \cos 2x > -1 \quad / : 2$$

$$\frac{1}{2} \sin 2x + \frac{\sqrt{3}}{2} \cos 2x > -\frac{1}{2}$$

$$\cos \frac{\pi}{3} \sin 2x + \sin \frac{\pi}{3} \cos 2x > -\frac{1}{2}$$

$$\sin \left(2x + \frac{\pi}{3} \right) > -\frac{1}{2}; \quad \left(\sin \frac{7\pi}{6} = -\frac{1}{2} \right)$$



$$-\frac{\pi}{6} + 2k\pi \leq 2x + \frac{\pi}{3} \leq \frac{7\pi}{6} + 2k\pi$$

$$-\frac{\pi}{2} + 2k\pi \leq 2x \leq \frac{5\pi}{6} + 2k\pi$$

$$-\frac{\pi}{4} + k\pi \leq x \leq \frac{5\pi}{12} + k\pi$$

8) $\sin x \cdot \sin 2x - \cos x \cdot \cos 2x > \sin 6x$

$$-\cos(2x + x) > \sin 6x$$

$$-\cos 3x - \sin 6x > 0 \quad / \cdot (-1)$$

$$\cos 3x + \sin 6x < 0$$

$$\cos 3x + 2 \sin 3x \cos 3x < 0$$

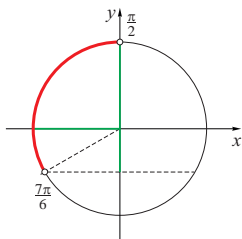
$$\cos 3x(1 + 2 \sin 3x) < 0$$

1° $\cos 3x < 0$

$$1 + 2 \sin 3x > 0$$

$$\cos 3x < 0$$

$$\sin 3x > -\frac{1}{2}$$



$$\frac{\pi}{2} + 2k\pi < 3x < \frac{7\pi}{6} + 2k\pi$$

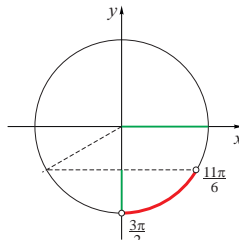
$$\frac{\pi}{6} + \frac{2k\pi}{3} < x < \frac{7\pi}{18} + \frac{2k\pi}{3}$$

2° $\cos 3x > 0$

$$1 + 2 \sin 3x < 0$$

$$\cos 3x > 0$$

$$\sin 3x < -\frac{1}{2}$$



$$\frac{3\pi}{2} + 2k\pi < 3x < \frac{11\pi}{6} + 2k\pi$$

$$\frac{\pi}{2} + \frac{2k\pi}{3} < x < \frac{11\pi}{18} + \frac{2k\pi}{3}$$