Zadatak 9.

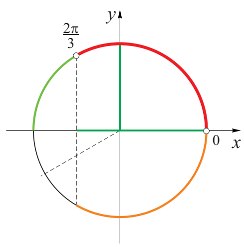
1) $\sin x + \sin 2x \geqslant 0$;
2) $\cos x + \cos 5x \geqslant 0$;
3) $\cos x + \cos 2x + \cos 3x > 0$;
4) $\cos 2x \leqslant \cos 3x - \cos 4x$;
5) $2\sin x \cdot \sin 2x \cdot \sin 3x < \sin 4x$;
6) $\sin x \cdot \cos 5x < \sin 9x \cdot \cos 3x$;
7) $\sin x \cdot \sin 3x > \sin 5x \cdot \sin 7x$;
8) $\sin x + \sin 3x < \sin 5x + \sin 7x$.

Rješenje. **1)**

$$\sin x + \sin 2x \geqslant 0$$
$$\sin x + 2\sin x\cos x \geqslant 0$$
$$\sin x(2\cos x + 1) \geqslant 0$$

1°
$$\begin{aligned}&\sin x \geqslant 0\\ &2\cos x + 1 \geqslant 0\end{aligned}$$

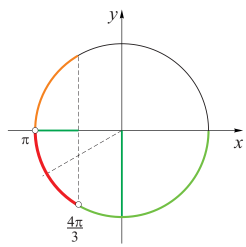
$$\sin x \geqslant 0$$
$$\cos x \geqslant -\frac{1}{2}$$



$$2k\pi < x < \frac{2\pi}{3} + 2k\pi$$

2°
$$\begin{aligned}&\sin x \leqslant 0\\ &2\cos x + 1 \leqslant 0\end{aligned}$$

$$\sin x \leqslant 0$$
$$\cos x \leqslant -\frac{1}{2}$$



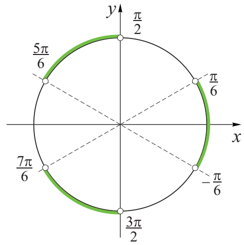
$$\pi + 2k\pi < x < \frac{4\pi}{3} + 2k\pi$$

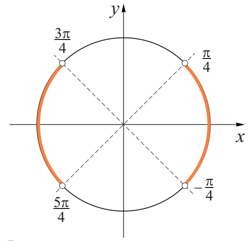
2)

$$\cos x + \cos 5x \geqslant 0$$
$$2\cos\frac{x+5x}{2}\cos\frac{x-5x}{2} \geqslant 0$$
$$\cos 3x \cos 2x \geqslant 0$$

1° $\cos 3x \geqslant 0$

$\cos 2x \geqslant 0$

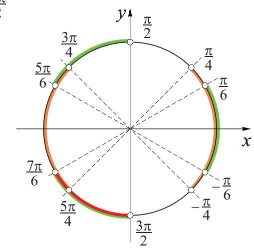




$$-\frac{\pi}{2} + 2k\pi \leqslant 3x \leqslant \frac{\pi}{2} + 2k\pi$$
$$-\frac{\pi}{2} + 2k\pi \leqslant 2x \leqslant \frac{\pi}{2} + 2k\pi$$

$$-\frac{\pi}{6} + \frac{2k\pi}{3} \leqslant x \leqslant \frac{\pi}{6} + \frac{2k\pi}{3}$$
$$-\frac{\pi}{4} + k\pi \leqslant x \leqslant \frac{\pi}{4} + k\pi$$

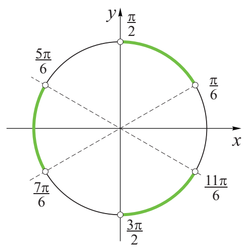
$\Rightarrow$

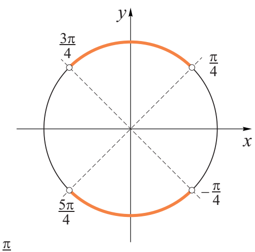


$$\Longrightarrow\ x \in \left\langle -\frac{\pi}{6} + k\pi, \frac{\pi}{6} + k\pi \right\rangle \cup \left\langle \frac{3\pi}{4} + k\pi, \frac{5\pi}{6} + k\pi \right\rangle \cup \left\langle \frac{7\pi}{6} + k\pi, \frac{3\pi}{2} + k\pi \right\rangle$$

2° $\cos 3x \leqslant 0$

$\cos 2x \leqslant 0$

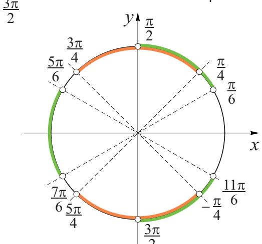




$$\frac{\pi}{2} + 2k\pi \leqslant 3x \leqslant \frac{3\pi}{2} + 2k\pi$$
$$\frac{\pi}{2} + 2k\pi \leqslant 2x \leqslant \frac{3\pi}{2} + 2k\pi$$

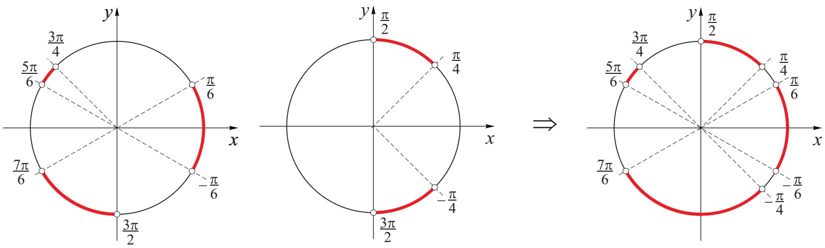
$$\frac{\pi}{6} + \frac{2k\pi}{3} \leqslant x \leqslant \frac{\pi}{2} + \frac{2k\pi}{3}$$
$$\frac{\pi}{4} + k\pi \leqslant x \leqslant \frac{3\pi}{4} + k\pi$$

$\Rightarrow$



$$\Longrightarrow\ x \in \left\langle \frac{\pi}{4} + k\pi, \frac{\pi}{2} + k\pi \right\rangle \cup \left\langle \frac{3\pi}{2} + k\pi, \frac{11\pi}{4} + k\pi \right\rangle$$

Unija rješenja:



$$\left\langle -\frac{\pi}{6}+k\pi, \frac{\pi}{6}+k\pi \right\rangle \cup \left\langle \frac{\pi}{4}+k\pi, \frac{\pi}{2}+k\pi \right\rangle \cup \left\langle \frac{3\pi}{4}+k\pi, \frac{5\pi}{6}+k\pi \right\rangle \cup \left\langle \frac{7\pi}{6}+k\pi, \frac{7\pi}{6}+k\pi \right\rangle$$

3)

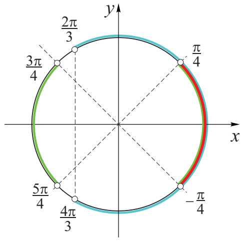
$$\cos x + \cos 2x + \cos 3x > 0$$
$$2\cos\frac{x+3x}{2}\cos\frac{x-3x}{2} + \cos 2x > 0$$
$$2\cos 2x\cos(-x) + \cos 2x > 0$$
$$\cos 2x(2\cos x + 1) > 0$$

1° $\quad \cos 2x > 0$

$2\cos x + 1 > 0$

$$\cos 2x > 0 \implies -\frac{\pi}{2} + 2k\pi < 2x < \frac{\pi}{2} + 2k\pi, \quad -\frac{\pi}{4} + k\pi < x < \frac{\pi}{4} + k\pi$$

$$2\cos x + 1 > 0, \quad \cos x > -\frac{1}{2} \implies -\frac{4\pi}{3} + 2k\pi < x < \frac{2\pi}{3} + 2k\pi$$



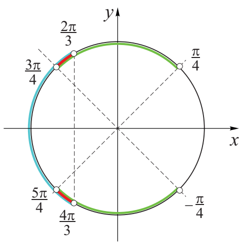
$$\implies -\frac{\pi}{4} + 2k\pi < x < \frac{\pi}{4} + 2k\pi$$

2° $\cos 2x < 0$

$2\cos x + 1 < 0$

$\cos 2x < 0 \implies \frac{\pi}{2} + 2k\pi < 2x < \frac{3\pi}{2} + 2k\pi, \quad \frac{\pi}{4} + k\pi < x < \frac{3\pi}{4} + k\pi$

$2\cos x + 1 < 0, \quad \cos x < -\frac{1}{2} \implies \frac{2\pi}{3} + 2k\pi < x < \frac{4\pi}{3} + 2k\pi$



$\implies \frac{2\pi}{3} + 2k\pi < x < \frac{3\pi}{4} + 2k\pi \text{ ili } \frac{5\pi}{4} + 2k\pi < x < \frac{4\pi}{3} + 2k\pi$

Unija rješenja: $-\frac{\pi}{4} + k\cdot 2\pi < x < \frac{\pi}{4} + k\cdot 2\pi$ ili $\frac{2\pi}{3} + k\cdot 2\pi < x < \frac{3\pi}{4} + k\cdot 2\pi$

ili $\frac{5\pi}{4} + k\cdot 2\pi < x < \frac{4\pi}{3} + k\cdot 2\pi$, $k \in \mathbf{Z}$;

4)

$\cos 2x \leqslant \cos 3x - \cos 4x$

$\cos 2x + \cos 4x - \cos 3x \leqslant 0$

$2\cos\frac{2x + 4x}{2}\cos\frac{2x - 4x}{2} - \cos 3x \leqslant 0$

$2\cos 3x \cos(-x) - \cos 3x \leqslant 0$

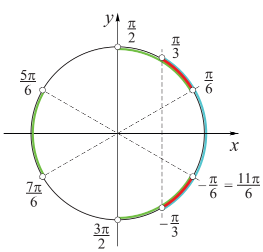
$\cos 3x(2\cos x - 1) \leqslant 0$

1° $\cos 3x \leqslant 0$

$2\cos x - 1 \geqslant 0$

$\cos 3x \leqslant 0 \implies \frac{\pi}{2} + 2k\pi \leqslant 3x \leqslant \frac{3\pi}{2} + 2k\pi, \quad \frac{\pi}{6} + \frac{2k\pi}{3} \leqslant x \leqslant \frac{\pi}{2} + \frac{2k\pi}{3}$

$2\cos x + 1 \geqslant 0, \quad \cos x \geqslant \frac{1}{2} \implies -\frac{\pi}{3} + 2k\pi \leqslant x \leqslant \frac{\pi}{3} + 2k\pi$



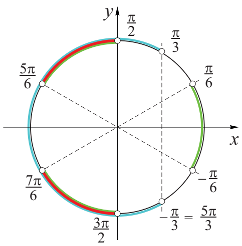
$\implies -\frac{\pi}{3} + 2k\pi \leqslant x \leqslant -\frac{\pi}{6} + 2k\pi \text{ ili } \frac{\pi}{6} + 2k\pi \leqslant x \leqslant \frac{\pi}{3} + 2k\pi$

2° $\cos 3x \geqslant 0$

$2\cos x - 1 \leqslant 0$

$$\cos 3x \geqslant 0 \implies -\frac{\pi}{2} + 2k\pi \leqslant 3x \leqslant \frac{\pi}{2} + 2k\pi, \quad -\frac{\pi}{6} + \frac{2k\pi}{3} \leqslant x \leqslant \frac{\pi}{6} + \frac{2k\pi}{3}$$

$$2\cos x + 1 \leqslant 0, \quad \cos x \leqslant \frac{1}{2} \implies \frac{\pi}{3} + 2k\pi \leqslant x \leqslant \frac{5\pi}{3} + 2k\pi$$



$$\implies \frac{\pi}{2} + 2k\pi \leqslant x \leqslant \frac{5\pi}{6} + 2k\pi \ \text{ ili } \ \frac{7\pi}{6} + 2k\pi \leqslant x \leqslant \frac{3\pi}{2} + 2k\pi$$

Unija rješenja: $-\frac{5\pi}{6} + k\cdot 2\pi \leqslant x \leqslant -\frac{\pi}{2} + k\cdot 2\pi$ ili $-\frac{\pi}{3} + k\cdot 2\pi \leqslant x \leqslant -\frac{\pi}{6} + k\cdot 2\pi$ ili $\frac{\pi}{6} + k\cdot 2\pi \leqslant x \leqslant \frac{\pi}{3} + k\cdot 2\pi$ ili $\frac{\pi}{2} + k\cdot 2\pi \leqslant x \leqslant \frac{5\pi}{6} + k\cdot 2\pi$, $k \in \mathbf{Z}$;

5)

$$2\sin x \cdot \sin 2x \cdot \sin 3x < \sin 4x$$
$$2\sin x \cdot \sin 2x \cdot \sin 3x - 2\sin 2x \cos 2x < 0$$
$$2\sin 2x \cdot (\sin x \cdot \sin 3x - \cos 2x) < 0$$
$$2\sin 2x \cdot \left[\frac{1}{2}(\cos 4x - \cos 2x) - \cos 2x\right] < 0$$
$$2\sin 2x \cdot \left(\frac{1}{2}\cos 4x - \frac{3}{2}\cos 2x\right) < 0$$
$$\sin 2x \cdot (\cos 4x - 3\cos 2x) < 0$$
$$\sin 2x \cdot (2\cos^2 2x - 1 - 3\cos 2x) < 0$$
$$\sin 2x \cdot (2\cos^2 2x - 3\cos 2x - 1) < 0$$

$$k\cdot\pi < x < \frac{\pi}{6} + k\cdot\pi \ \text{ ili } \ \frac{\pi}{2} + k\cdot\pi < x < \frac{5\pi}{6} + k\cdot\pi, \ k \in \mathbf{Z};$$

6)

$$\sin x \cdot \cos 5x < \sin 9x \cdot \cos 3x$$
$$\frac{1}{2}[\sin(x+5x)+\sin(x-5x)] < \frac{1}{2}[\sin(9x+3x)+\sin(9x-3x)]$$
$$\sin 6x + \sin(-4x) < \sin 12x + \sin 6x$$
$$\sin 12x + \sin 4x > 0$$
$$2\sin\frac{4x+12x}{2}\cos\frac{4x-12x}{2} > 0$$
$$\sin 8x \cdot \cos 4x > 0$$
$$2\sin 4x \cdot \cos 4x \cdot \cos 4x > 0$$
$$\sin 4x \cdot \underbrace{\cos^2 4x}_{\geqslant 0} > 0$$
$$\cos 4x \neq 0 \implies 4x \neq \frac{\pi}{2} + k\pi, \quad \underline{x \neq \frac{\pi}{8} + \frac{k\pi}{4}}$$
$$\sin 4x > 0 \implies 2k\pi < 4x < (2k+1)\pi, \quad \underline{\frac{k\pi}{2} < x < \frac{(2k+1)\pi}{4}}, \quad k \in \mathbf{Z}$$

Konačno rješenje:

$$k\cdot\frac{\pi}{2} < x < \frac{\pi}{8} + k\cdot\frac{\pi}{2} \quad \text{ili} \quad \frac{\pi}{8} + k\cdot\frac{\pi}{2} < x < \frac{\pi}{4} + k\cdot\frac{\pi}{2}, \quad k \in \mathbf{Z};$$

7)

$$\sin x \sin 3x > \sin 5x \sin 7x$$
$$\frac{1}{2}[\cos(x-3x)-\cos(x+3x)] > \frac{1}{2}[\cos(5x-7x)-\cos(5x+7x)]$$
$$\cos 2x - \cos 4x > \cos 2x - \cos 12x$$
$$\cos 12x - \cos 4x > 0$$
$$-2\sin\frac{12x+4x}{2}\sin\frac{12x-4x}{2} > 0$$
$$\sin 8x \sin 4x < 0$$
$$2\sin 4x \cos 4x \sin 4x < 0$$
$$\sin^2 4x \cos 4x < 0$$
$$\sin 4x \neq 0 \implies 4x \neq k\pi, \quad \underline{x \neq \frac{k\pi}{4}}, \quad k \in \mathbf{Z}$$
$$\cos 4x < 0 \implies \frac{\pi}{2} + 2k\pi < 4x < \frac{3\pi}{2} + 2k\pi, \quad \underline{\frac{\pi}{8} + \frac{k\pi}{2} < x < \frac{3\pi}{8} + \frac{k\pi}{2}}, \quad k \in \mathbf{Z}$$

$$\frac{\pi}{8} + k\cdot\frac{\pi}{2} < x < \frac{\pi}{4} + k\cdot\frac{\pi}{2} \quad \text{ili} \quad \frac{\pi}{4} + k\cdot\frac{\pi}{2} < x < \frac{3\pi}{8} + k\cdot\frac{\pi}{2}, \quad k \in \mathbf{Z};$$

8)

$$\sin x + \sin 3x < \sin 5x + \sin 7x$$

$$2\sin\frac{x+3x}{2}\cos\frac{x-3x}{2} < 2\sin\frac{5x+7x}{2}\cos\frac{5x-7x}{2} \quad / : 2$$

$$\sin 2x\cos(-x) < \sin 6x\cos(-x)$$

$$\sin 6x\cos x - \sin 2x\cos x > 0$$

$$\cos x(\sin 6x - \sin 2x) > 0$$

$$\cos x \cdot 2\cos\frac{6x+2x}{2}\sin\frac{6x-2x}{2} > 0 \quad / : 2$$

$$\cos x\cos 4x\sin 2x > 0$$

$$\cos x\cos 4x\cos x\sin x > 0$$

$$\sin x\cos 4x\underbrace{\cos^2 x}_{\geqslant 0} > 0$$

$$\cos x \neq 0 \implies \underline{x \neq \frac{2k+1}{2}\pi}, \quad k \in \mathbf{Z}$$

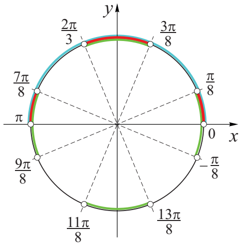
$$\sin x\cos 4x > 0, \quad x \neq \frac{2k+1}{2}\pi$$

1° $\sin x > 0$

$\cos 4x > 0$

$$\sin x > 0 \implies 2k\pi < x < (2k+1)\pi$$

$$\cos 4x > 0 \implies -\frac{\pi}{2} + 2k\pi < 4x < \frac{\pi}{2} + 2k\pi, \quad -\frac{\pi}{8} + \frac{k\pi}{2} < x < \frac{\pi}{8} + \frac{k\pi}{2}$$



$$\implies 2k\pi < x < \frac{\pi}{8} + 2k\pi \quad \text{ili}$$

$$\frac{3\pi}{8} + 2k\pi < x < \frac{\pi}{2} + 2k\pi \quad \text{ili}$$

$$\frac{\pi}{2} + 2k\pi < x < \frac{5\pi}{8} + 2k\pi \quad \text{ili}$$

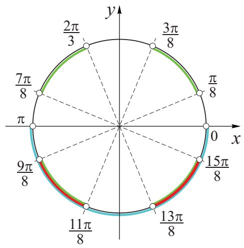
$$\frac{7\pi}{8} + 2k\pi < x < (2k+1)\pi + 2k\pi$$

2° $\sin x < 0$

$\cos 4x < 0$

$$\sin x < 0 \implies (2k-1)\pi < x < 2k\pi$$

$$\cos 4x < 0 \implies \frac{\pi}{2} + 2k\pi < 4x < \frac{3\pi}{2} + 2k\pi, \quad \frac{\pi}{8} + \frac{k\pi}{2} < x < \frac{3\pi}{8} + \frac{k\pi}{2}$$



$$\implies \frac{9\pi}{8} + 2k\pi < x < \frac{11\pi}{8} + 2k\pi \quad \text{ili}$$

$$\frac{13\pi}{8} + 2k\pi < x < \frac{15\pi}{8} + 2k\pi\,.$$

Sva rješenja su:

$$k \cdot 2\pi < x < \frac{\pi}{8} + k \cdot 2\pi \ \text{ ili } \ \frac{3\pi}{8} + k \cdot 2\pi < x < \frac{\pi}{2} + k \cdot 2\pi \ \text{ ili}$$

$$\frac{\pi}{2} + k \cdot 2\pi < x < \frac{5\pi}{8} + k \cdot 2\pi \ \text{ ili } \ \frac{7\pi}{8} + k \cdot 2\pi < x < (2k+1)\pi \ \text{ ili}$$

$$\frac{9\pi}{8} + k \cdot 2\pi < x < \frac{11\pi}{8} + k \cdot 2\pi \ \text{ ili } \ \frac{13\pi}{8} + k \cdot 2\pi < x < \frac{15\pi}{8} + k \cdot 2\pi\,, \ k \in \mathbf{Z}\,.$$