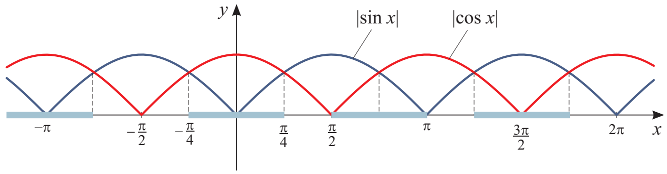
Zadatak 10. **1)** $|\sin x| < |\cos x|$; **2)** $|\sin x| + |\cos x| > 1$;

3) $|\sin x| \cdot \cos x > \frac{1}{4}$; **4)** $\sin |x| \cdot \cos x > 0.25$;

5) $4(\sin^2 x - |\cos x|) < 1$;

Rješenje. **1)**



$$-\frac{\pi}{4} + k \cdot \pi < x < \frac{\pi}{4} + k \cdot \pi, \ k \in \mathbf{Z};$$

2)

$$|\sin x| + |\cos x| > 1 \quad /^2$$
$$|\sin x|^2 + |\cos x|^2 + 2|\sin x||\cos x| > 1$$
$$\sin^2 x + \cos^2 x + 2|\sin x \cdot \cos x| > 1$$
$$1 + 2|\sin x \cdot \cos x| > 1$$
$$2|\sin x \cdot \cos x| > 0 \quad /:2$$
$$|\sin 2x| > 0$$
$$\sin 2x \neq 0 \implies 2x \neq k\pi, \ x \neq \frac{k\pi}{2}$$
$$|\sin 2x| > 0, \ \forall x \in \mathbf{R}, \ x \neq \frac{k\pi}{2}, \ k \in \mathbf{Z}$$

3)

$$|\sin x| \cdot \cos x > \frac{1}{4}$$
$$|\sin x| = \begin{cases} \sin x & 2k\pi \leqslant x \leqslant (2k+1)\pi \\ -\sin x & (2k-1)\pi < x < 2k\pi \end{cases}$$

jednadžba može glasiti:

$1^\circ \quad 2k\pi \leqslant x \leqslant (2k+1)\pi$

$$\sin x \cos x > \frac{1}{4}$$
$$\frac{1}{2}\sin 2x > \frac{1}{4}$$
$$\sin 2x > \frac{1}{2}$$
$$\frac{\pi}{6} + 2k\pi < 2x < \frac{5\pi}{6} + 2k\pi \quad /:2$$
$$\frac{\pi}{12} + k\pi < x < \frac{5\pi}{12} + k\pi$$

$2^\circ \quad (2k-1)\pi < x < 2k\pi$

$$-\sin x \cos x > \frac{1}{4}$$
$$-\frac{1}{2}\sin 2x > \frac{1}{4}$$
$$\sin 2x < -\frac{1}{2}$$
$$\frac{7\pi}{6} + 2k\pi < 2x < \frac{11\pi}{6} + 2k\pi \quad /:2$$
$$\frac{7\pi}{12} + k\pi < x < \frac{11\pi}{12} + k\pi$$

4)

$$\sin|x| \cdot \cos x > 0.25$$

$$\sin|x| = \begin{cases} -\sin x & x < 0 \\ \sin x & x \geqslant 0 \end{cases}$$

jednadžba može glasiti:

$1^\circ \quad x < 0$

$$-\sin x \cos x > \frac{1}{4}$$

$$-\frac{1}{2}\sin 2x > \frac{1}{4}$$

$$\sin 2x < -\frac{1}{2}$$

$$-\frac{5\pi}{6} + 2k\pi < 2x < -\frac{\pi}{6} + 2k\pi \quad / : 2$$

$$-\frac{5\pi}{12} + 2k\pi < x < -\frac{\pi}{12} + 2k\pi$$

uz uvjet $x < 0$ rješenje je

$$-\frac{5\pi}{12} + 2k\pi < x < -\frac{\pi}{12} + 2k\pi,$$

$$k = 0, -1, -2, \ldots$$

$2^\circ \quad x \geqslant 0$

$$\sin x \cos x > \frac{1}{4}$$

$$\frac{1}{2}\sin 2x > \frac{1}{4}$$

$$\sin 2x > \frac{1}{2}$$

$$\frac{\pi}{6} + 2k\pi < 2x < \frac{5\pi}{6} + 2k\pi \quad / : 2$$

$$\frac{\pi}{12} + k\pi < x < \frac{5\pi}{12} + k\pi$$

uz uvjet $x > 0$ rješenje je

$$\frac{\pi}{12} + k\pi < x < \frac{5\pi}{12} + k\pi,$$

$$k = 0, 1, 2, \ldots$$

5)

$$4(\sin^2 x - |\cos x|) < 1$$

$$|\cos x| = \begin{cases} \cos x & -\frac{\pi}{2} + 2k\pi \leqslant x \leqslant \frac{\pi}{2} + 2k\pi \\ -\cos x & \frac{\pi}{2} + 2k\pi \leqslant x \leqslant \frac{3\pi}{2} + 2k\pi \end{cases}$$

jednadžba može glasiti:

1° $-\frac{\pi}{2} + 2k\pi \leqslant x \leqslant \frac{\pi}{2} + 2k\pi$

$$4(\sin^2 x - \cos x) < 1$$

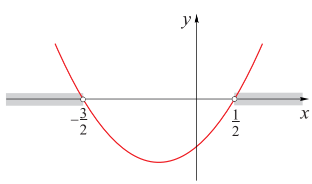
$$4(1 - \cos^2 x - \cos x) < 1$$

$$4 - 4\cos^2 x - 4\cos x - 1 < 0 \quad / \cdot (-1)$$

$$4\cos^2 x + 4\cos x - 3 < 0$$

$$(\cos x)_{1,2} = \frac{-4 \pm \sqrt{16 + 48}}{8} = \frac{-4 \pm 8}{8}$$

$$(\cos x)_1 = -\frac{3}{2}, \quad (\cos x)_2 = \frac{1}{2}$$



$(\cos x)_1 < -\frac{3}{2}$ nije rješenje

$$(\cos x)_2 > \frac{1}{2} \implies -\frac{\pi}{3} + 2k\pi < x < \frac{\pi}{3} + 2k\pi$$

2° $\frac{\pi}{2} + 2k\pi \leqslant x \leqslant \frac{3\pi}{2} + 2k\pi$

$$4(\sin^2 x + \cos x) < 1$$

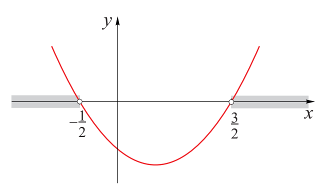
$$4(1 - \cos^2 x + \cos x) < 1$$

$$4 - 4\cos^2 x + 4\cos x - 1 < 0 \quad / \cdot (-1)$$

$$4\cos^2 x - 4\cos x - 3 < 0$$

$$(\cos x)_{1,2} = \frac{4 \pm \sqrt{16 + 48}}{8} = \frac{4 \pm 8}{8}$$

$$(\cos x)_1 = \frac{3}{2}, \quad (\cos x)_2 = -\frac{1}{2}$$



$(\cos x)_1 > \frac{3}{2}$ nije rješenje

$$(\cos x)_2 < -\frac{1}{2} \implies \frac{2\pi}{3} + 2k\pi < x < \frac{4\pi}{3} + 2k\pi$$

Unija rješenja pod 1° i 2°:

$$-\frac{\pi}{3} + k \cdot \pi < x < \frac{\pi}{3} + k \cdot \pi, \quad k \in \mathbf{Z};$$