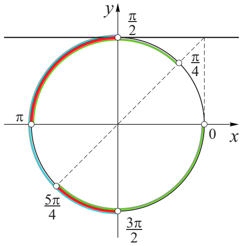
Zadatak 11. **1)** $\sin x+\cos x<\dfrac{1}{\sin x}$;

2) $\sin x+\cos x<\dfrac{1}{\cos x}$.

Rješenje. **1)** Nejednadžbu možemo sljedećim postupkom svesti na jednostavniji oblik:

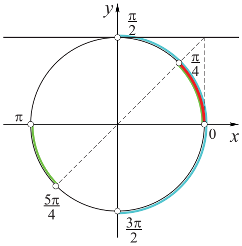
$$\sin x+\cos x<\frac{1}{\sin x}$$
$$\sin x+\cos x-\frac{1}{\sin x}<0$$
$$\frac{\sin^2 x+\cos x\sin x-1}{\sin x}<0$$
$$\frac{\cos x\sin x-\cos^2 x}{\sin x}<0$$
$$\frac{\cos x(\sin x-\cos x)}{\sin x}<0$$
$$\cos x(1-\operatorname{ctg} x)<0$$

1° $\cos x<0$, $1-\operatorname{ctg} x>0$ ⟶ $\cos x<0$, $\operatorname{ctg} x<1$



$$\Longrightarrow \frac{\pi}{2}+2k\pi<x<(k+1)\pi$$
$$\text{ili } \frac{5\pi}{4}+2k\pi<x<\frac{3\pi}{2}+2k\pi$$

2° $\cos x>0$, $1-\operatorname{ctg} x<0$ ⟶ $\cos x>0$, $\operatorname{ctg} x>1$



$$\Longrightarrow 2k\pi<x<\frac{\pi}{4}+2k\pi$$

Rješenje je:

$$\frac{5\pi}{4}+k\cdot 2\pi<x<\frac{3\pi}{2}+k\cdot 2\pi \text{ ili } k\cdot 2\pi<x<\frac{\pi}{4}+k\cdot 2\pi \text{ ili } \frac{\pi}{2}+k\cdot 2\pi<x<(2k+1)\pi,\ k\in\mathbf{Z}.$$

2) Nejednadžbu možemo sljedećim postupkom svesti na jednostavniji oblik:

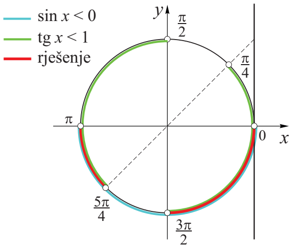
$$\sin x + \cos x < \frac{1}{\cos x}$$
$$\sin x + \cos x - \frac{1}{\cos x} < 0$$
$$\frac{\sin x \cos x + \cos^2 x - 1}{\cos x} < 0$$
$$\frac{\sin x \cos x - \sin^2 x}{\cos x} < 0$$
$$\frac{\sin x(\cos x - \sin x)}{\cos x} < 0$$
$$\sin x(1 - \operatorname{tg} x) < 0$$

1° $\sin x < 0$

$1 - \operatorname{tg} x > 0$

$\sin x < 0$

$\operatorname{tg} x < 1$



$$\Longrightarrow \quad (2k+1)\pi < x < \frac{5\pi}{4} + 2k\pi$$

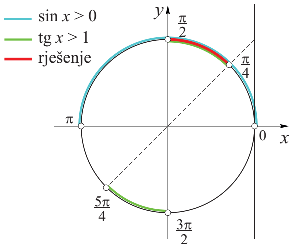
ili $\quad -\dfrac{\pi}{2} + 2k\pi < x < 2k\pi$

2° $\sin x > 0$

$1 - \operatorname{tg} x < 0$

$\sin x > 0$

$\operatorname{tg} x > 1$



$$\Longrightarrow \quad \frac{\pi}{4} + 2k\pi < x < \frac{\pi}{2} + 2k\pi$$

Rješenje je:

$$-\frac{\pi}{2} + k\cdot 2\pi < x < k\cdot 2\pi \text{ ili } \frac{\pi}{4} + k\cdot 2\pi < x < \frac{\pi}{2} + k\cdot 2\pi \text{ ili } (2k+1)\pi < x < \frac{5\pi}{4} + k\cdot 2\pi,\ k \in \mathbf{Z}.$$