Rješenja složenijih zadataka

Zadatak 1. $\sin^4 x + \cos^4 x = \sin^4 2x + \cos^4 2x\,.$

Rješenje. Jednadžbu zapišimo u obliku

$$\sin^4 x + \cos^4 x = \sin^4 2x + \cos^4 2x$$
$$(\sin^2 x + \cos^2 x)^2 - 2\sin^2 x\cos^2 x = (\sin^2 2x + \cos^2 2x)^2 - 2\sin^2 2x\cos^2 2x \quad / \cdot (-2)$$
$$4\sin^2 x\cos^2 x = 4\sin^2 2x\cos^2 2x$$
$$\sin^2 2x = 4\sin^2 2x\cos^2 2x$$
$$\sin^2 2x(1 - 4\cos^2 2x) = 0$$

1° $\sin^2 2x = 0$

$\sin 2x = 0$

$2x = k\pi$

$x = k\cdot\dfrac{\pi}{2}$

2° $1 - 4\cos^2 2x = 0$

$4\cos^2 2x = 1$

$\cos^2 2x = \dfrac{1}{4}$

$(\cos 2x)_{1,2} = \pm\dfrac{1}{2}$

$2x_{1,2} = \pm\dfrac{\pi}{3} + k\pi$

$x_{1,2} = \pm\dfrac{\pi}{6} + k\cdot\dfrac{\pi}{2}$

Rješenje jednadžbe je $x = k\cdot\dfrac{\pi}{2}$ ili $x = \pm\dfrac{\pi}{6} + k\cdot\dfrac{\pi}{2}$, $k \in \mathbf{Z}$.