

Zadatak 2. $\sqrt{2}(\sin^3 x + \cos^3 x) = \sin 2x.$

Rješenje. Zapišimo jednadžbu u obliku

$$\sqrt{2}(\sin^3 x + \cos^3 x) = \sin 2x$$

$$\sqrt{2} \cdot (\sin x + \cos x) \left(1 - \frac{1}{2} \sin 2x\right) = \sin 2x \quad / \cdot \sqrt{2}$$

$$(\sin x + \cos x)(2 - \sin 2x) = \sqrt{2} \cdot \sin 2x \quad /^2$$

$$(\sin^2 x + \cos^2 x + 2 \sin x \cos x)(4 + \sin^2 2x - 4 \sin 2x) = 2 \sin^2 2x$$

$$(1 + \sin 2x)(4 + \sin^2 2x - 4 \sin 2x) = 2 \sin^2 2x$$

$$\sin^3 2x - 5 \sin^2 2x + 4 = 0$$

$$\sin^3 2x - \sin^2 2x - 4 \sin^2 2x + 4 = 0$$

$$\sin^2 2x(\sin 2x - 1) - 4(\sin^2 2x - 1) = 0$$

$$\sin^2 2x(\sin 2x - 1) - 4(\sin 2x - 1)(\sin 2x + 1) = 0$$

$$(\sin 2x - 1)(\sin^2 2x - 4 \sin 2x - 4) = 0$$

$$1^\circ \quad \sin 2x - 1 = 0$$

$$\sin 2x = 1$$

$$2x = \frac{\pi}{2} + 2k\pi$$

$$x_1 = \frac{\pi}{4} + k\pi$$

$$2^\circ \quad \sin^2 2x - 4 \sin 2x - 4 = 0$$

$$(\sin 2x)_{1,2} = \frac{4 \pm \sqrt{16 + 16}}{2} = \frac{4 \pm 4\sqrt{2}}{2} = 2(1 \pm \sqrt{2})$$

$$\sin 2x = 2(1 - \sqrt{2}) \approx -0.82843$$

(drugo rješenje otpada jer je > 1)

$$2x_2 = \arcsin(-0.82843)$$

$$x_2 = \frac{1}{2} \arcsin(-0.82843)$$