

**Zadatak 9.** Koliko je  $\sin(\alpha + \beta)$  i  $\cos(\alpha + \beta)$  ako je  $\operatorname{tg} \alpha = \frac{8}{15}$ ,  $\operatorname{ctg} \beta = -\frac{7}{24}$ , te  $0 < \alpha < \frac{\pi}{2}$ ,  $\frac{\pi}{2} < \beta < \pi$ ?

**Rješenje.**  $\operatorname{tg} \alpha = \frac{8}{15}$ ,  $0 < \alpha < \frac{\pi}{2} \implies \sin \alpha, \cos \alpha, \operatorname{ctg} \alpha > 0$

$\operatorname{ctg} \beta = -\frac{7}{24}$ ,  $\frac{\pi}{2} < \beta < \pi \implies \sin \beta > 0, \cos \beta, \operatorname{tg} \beta < 0$ :

$$\cos \alpha = \sqrt{\frac{1}{1 + \operatorname{tg}^2 \alpha}} = \sqrt{\frac{1}{1 + \frac{64}{225}}} = \sqrt{\frac{1}{\frac{289}{225}}} = \sqrt{\frac{225}{289}} = \frac{15}{17}$$

$$\sin \alpha = \sqrt{1 - \cos^2 \alpha} = \sqrt{1 - \frac{225}{289}} = \frac{8}{17}$$

$$\sin \beta = \sqrt{\frac{1}{1 + \operatorname{ctg}^2 \beta}} = \sqrt{\frac{1}{1 + \frac{49}{576}}} = \sqrt{\frac{1}{\frac{625}{576}}} = \sqrt{\frac{576}{625}} = \frac{24}{25}$$

$$\cos \beta = -\sqrt{1 - \sin^2 \beta} = -\sqrt{1 - \frac{576}{625}} = -\frac{7}{25}$$

$$\sin(\alpha + \beta) = \frac{8}{17} \cdot \left(-\frac{7}{25}\right) + \frac{15}{17} \cdot \frac{24}{25} = -\frac{56}{425} + \frac{360}{425} = \frac{304}{425}$$

$$\cos(\alpha + \beta) = \frac{15}{17} \cdot \left(-\frac{7}{25}\right) - \frac{8}{17} \cdot \frac{24}{25} = \frac{-105 - 192}{425} = -\frac{297}{425}$$