**Zadatak 10.** Ako je $\cos\left(\frac{\pi}{2}-\alpha\right)=\frac{5}{13}$, $\sin\left(\frac{\pi}{2}-\beta\right)=\frac{3}{5}$, te $\frac{\pi}{2}<\alpha<\pi$, $0<\beta<\frac{\pi}{2}$, koliko je $\sin(\alpha+\beta)$ i $\sin(\alpha-\beta)$?

***Rješenje.*** $\frac{\pi}{2}<\alpha<\pi \implies \cos\alpha>0,\ \sin\alpha,\ \operatorname{tg}\alpha,\ \operatorname{ctg}\alpha<0$

$$0<\beta<\frac{\pi}{2} \implies \cos\beta,\ \sin\beta,\ \operatorname{tg}\beta,\ \operatorname{ctg}\beta>0$$

$$\cos\left(\frac{\pi}{2}-\alpha\right)=\underbrace{\cos\frac{\pi}{2}}_{=0}\cdot\cos\alpha+\underbrace{\sin\frac{\pi}{2}}_{=1}.\sin\alpha=\sin\alpha=\frac{5}{13} \implies \sin\alpha=\frac{5}{13}$$

$$\cos\alpha=-\sqrt{1-\sin^2\alpha}=-\sqrt{1-\frac{25}{169}}=-\frac{12}{13}$$

$$\sin\left(\frac{\pi}{2}-\beta\right)=\underbrace{\sin\frac{\pi}{2}}_{=1}\cos\beta-\underbrace{\cos\frac{\pi}{2}}_{=0}\sin\beta=\frac{3}{5} \implies \cos\beta=\frac{3}{5}$$

$$\sin\beta=\sqrt{1-\cos^2\beta}=\sqrt{1-\frac{9}{25}}=\frac{4}{5}$$

$$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta=\frac{5}{13}\cdot\frac{3}{5}+\left(-\frac{12}{13}\right)\cdot\frac{4}{5}=\frac{15-48}{65}=-\frac{33}{65}$$

$$\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta=\frac{5}{13}\cdot\frac{3}{5}-\left(-\frac{12}{13}\right)\cdot\frac{4}{5}=\frac{15+48}{65}=\frac{63}{65}$$